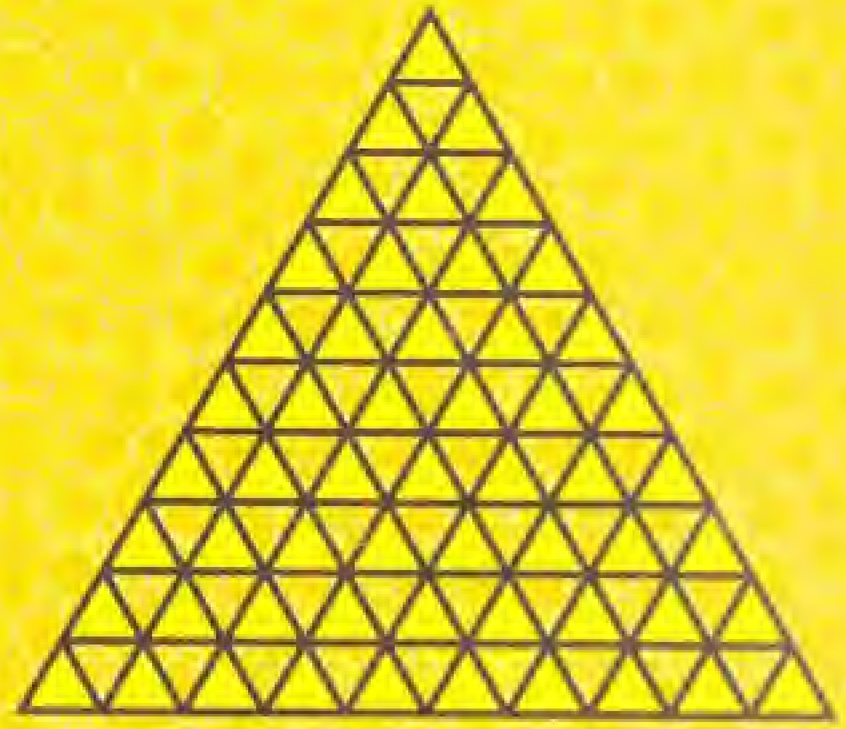

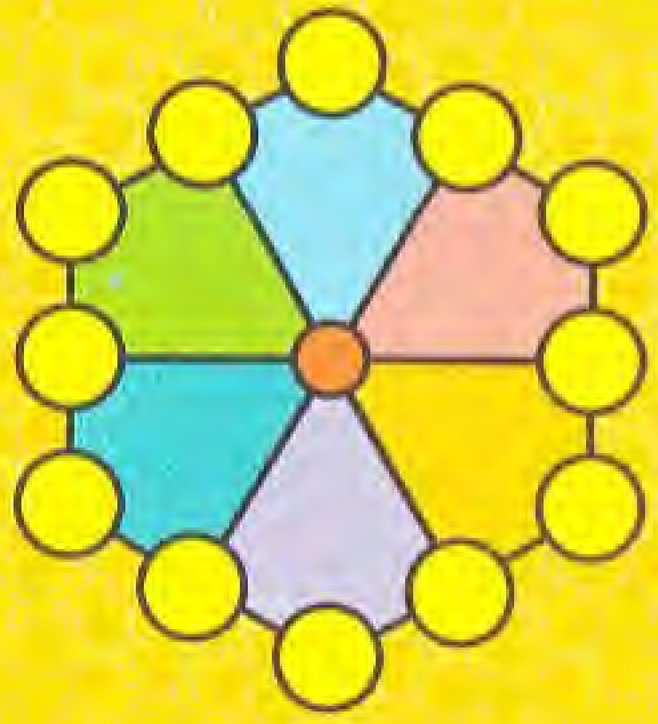

VIKRAM A SARABHAI
COMMUNITY SCIENCE CENTRE

Navrangpura, Ahmedabad - 380 009
T: +91 79 26302914, 26302085
E: info@vascsc.org, W: www.vascsc.org

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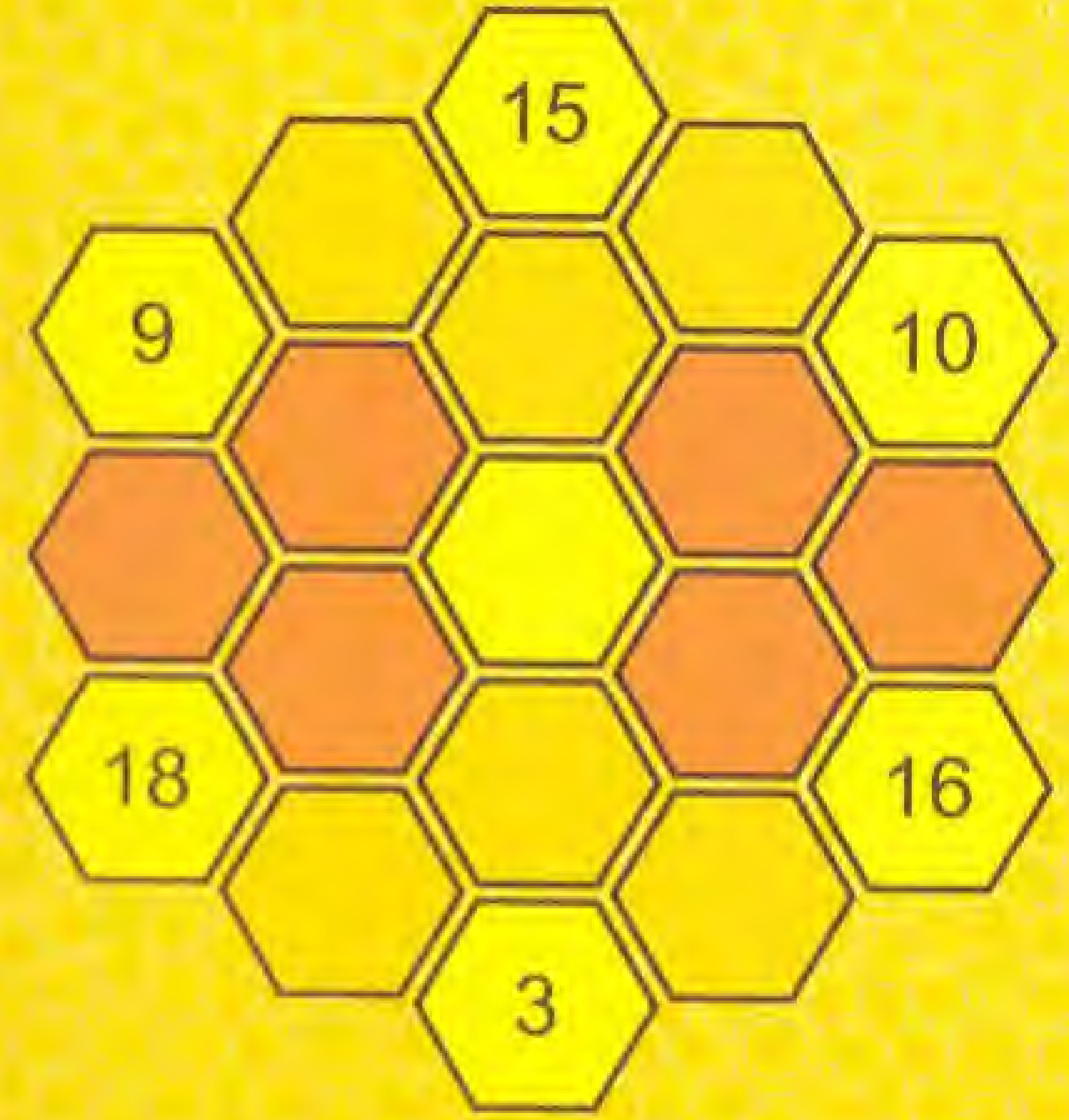
Brain Sharpeners



A. R. Rao

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E: info@vascsc.org, W: www.vascsc.org

FROM THE PREVIOUS EDITIONS

Problem solving besides being an enjoyable pastime, provides the solver an opportunity to develop skills and stimulate interest in mathematical and logical reasoning. Taken in a right spirit, they are a source of much fun and the mental effort involved in successfully solving them is a rewarding experience. A few of the problems appearing in this book are known, but they are included for those who will read them for the first time and experience the thrill of solving them. It is impossible, for the purpose of acknowledgment, to find out the original innovators of such problems.

The problems are a mixed variety, not graded according to difficulty. One may find somewhere a very easy problem and somewhere else a very difficult one. One might pick up a problem at random or select those that he might like to solve. Many people approach a problem in a spirit of challenge and that, we think, is the right attitude. So, the book has been divided into four sections. The first section contains only the statement of the problems. The second contains some hints for solution which should be referred to only if the solver is desperately in need of help. After reading the hints he should go back for a renewed attack on the problem. If he ultimately accepts defeat, he may read through the solution in the third section. It is hoped that the fourth section on comments may be enlightening and provocative of some new thoughts on a problem. Even if a reader solves a problem successfully it may be worth-while to read the hints, the solution and the comments on the problem to compare his own thoughts with those of the author.

A slightly better mathematical knowledge may be found necessary for the solution of some of the problems in the latter part of the book. In the case of a few of the puzzles, it may be advisable to engage oneself in some simple experimental work with thick papers, scissors and drawing instruments. Readers desirous of acquiring a mathematical laboratory type of experience may refer to my book: **A Manual of Mathematical Models and Teaching Aids,** wherein references to other materials and a bibliography are given.

An important realization in our institutions these days is the educational impact of problem solving. I am grateful to many readers for the interest they have evinced in this direction.

I will be greateful for reactions of the readers. Constructive suggestions for the improvement of the book are also welcome.

A. R. Rao
VASCSC

PREFACE TO THE FOURTH EDITION

We are happy to bring out the fourth edition of Brain Sharpeners to meet the demand of students, teachers, and others who have found the book as a source of not only recreation, but also of some non-formal education in Mathematics.

We have made some changes in a few problems, so as to make them relevant 'today'. We have also added quite a few new problems to the collection.

We continue to hold the view that the formal presentation of the problems, hints, solutions and comments in separate sections is an incentive.

We hope that the section of comments on many of the problems is stimulating and leads to some creative thinking.

I am thankful to Mrs. Hemaben Vasavada*, my colleague at the Centre, for her hard work pertaining to the revision and production of this edition.

Someone suggested that any one with a little knowledge of school mathematics could banish boredom, specially during long journeys, if the person has a few sheets of blank papers, a pencil and this book.

A. R. Rao
VASCSC

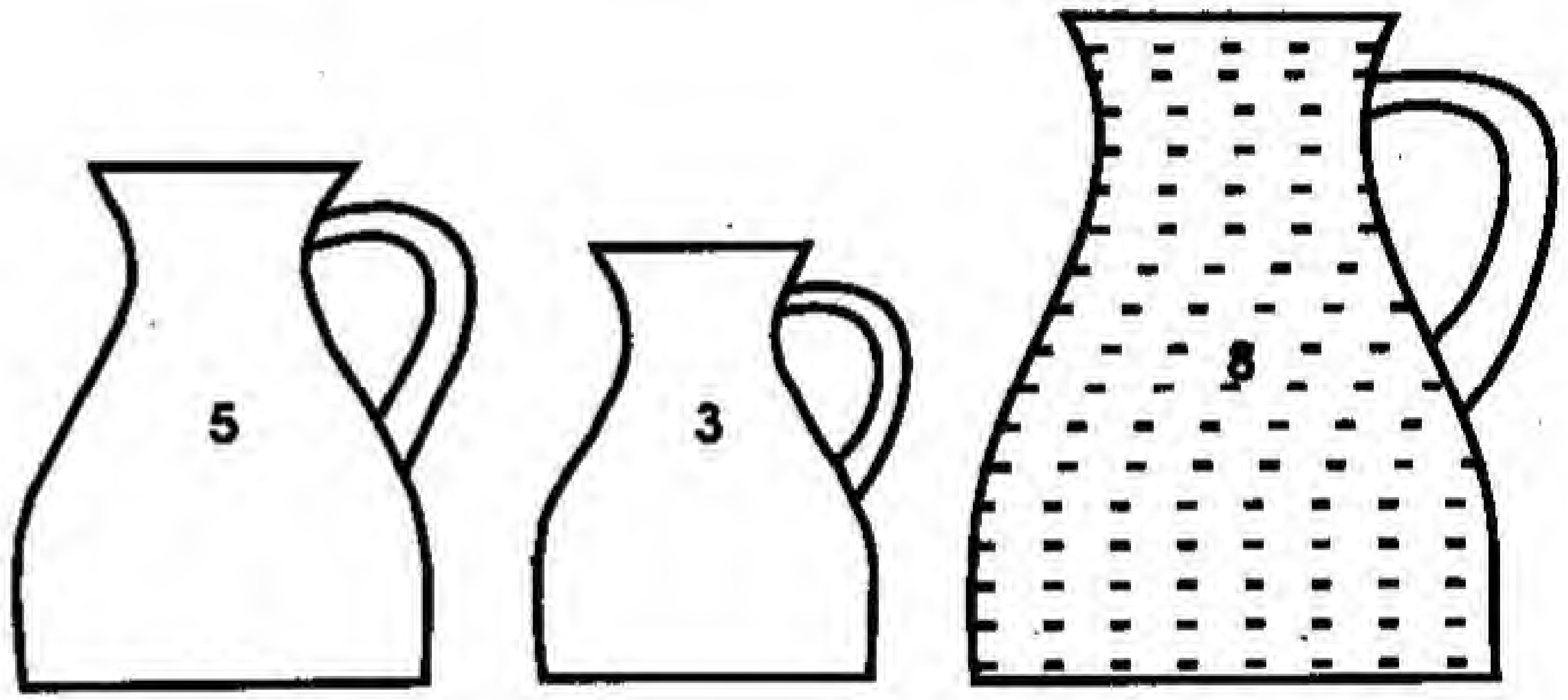
*Mrs. Hemaben Vasavada was a lecturer in Mathematics at the M.G.Science Institute, Ahmedabad and then at the M.B.Patel Science College, Anand. At present she is offering her services as subject expert at VASCSC,Ahmedabad. Her keen interest in mathematical laboratory was apparent since 1961, when she was associated with maths exhibition. She was the President of the Gujarat Ganit Mandal for the year 2007.

CONTENTS

PROBLEMS

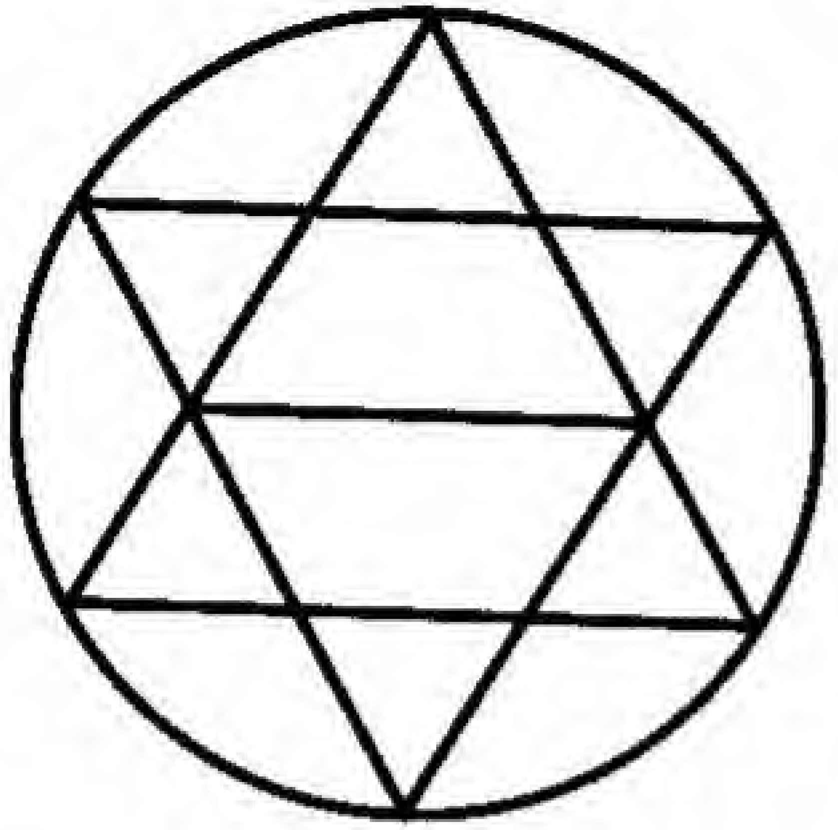
1. MILKMAN'S PROBLEM

A milkman has an eight litre jug full of milk and two empty jugs of capacities of five and three litres only. How can he divide the milk equally between two customers who should get four litres of milk each ?



Hints on p. 71 Solution on p. 103 Comments on p. 196

2. UNICURSAL FIGURE



Draw the figure without lifting the pencil from the paper and without retracing (repeating) any line.

Hints on p. 71 Solution on p. 104 Comments on p. 196

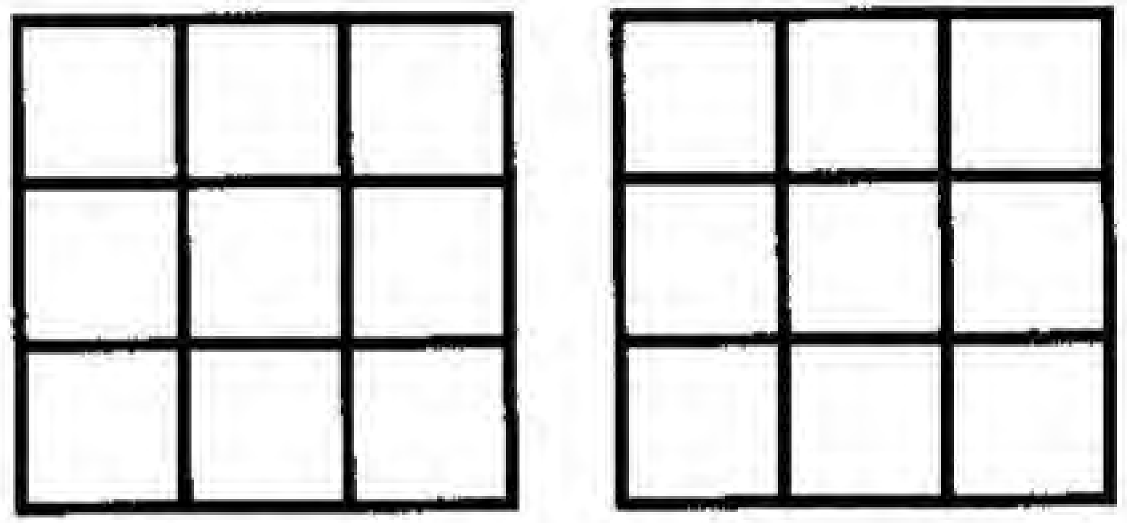
3. NECKLACES

Given plenty of black and red beads, how many different kinds of seven bead necklaces can you weave ?

Hints on p. 71 Solution on p. 105 Comments on p. 196

4. MAGIC SQUARES

In two different ways, fill up the nine squares below with different natural numbers (which may or may not be consecutive) such that the sum of the three numbers in each row, column or diagonal may always be 105.



Hints on p. 71 Solution on p. 106 Comments on p. 197

5. A RUPEE LOST

A hawker X sells 180 lemons in a vegetable market every day at the rate of 4 lemons per rupee and thus brings home Rs. 45. Another hawker Y, his neighbour, also sells 180 lemons every day at the rate of 5 lemons per rupee and brings home Rs. 36. One day, when Y fell sick, X offered to take his (Y's) lemons also to the market for selling. X then mixes all the lemons and sells 360 lemons at the rate of 9

lemons for two rupees, and thereby gets Rs. 80. He had to hand over Rs. 36 to Y in the evening and retained only Rs. 44 instead of Rs. 45 as his share. How did he lose one rupee?

Hints on p. 72 Solution on p. 106

6. ZEROLESS FACTORS

Write down two numbers whose product is 1000000000. The two numbers should not contain any zero.

Hints on p. 72 Solution on p.107 Comments on p. 197

7. FIND THE NUMBER

Find the smallest number which is divisible by 11 and

leaves reminder	when divided by
1	3
3	5
5	7
7	9

Hints on p. 72 Solution on p. 107 Comments on p. 197

8. AGE PROBLEM

Four years ago, my age was twice what my son's age will be four years hence. How many years hence will my age be twice my son's age, at the same time?

Hints on p. 72 Solution on p. 107 Comments on p. 197

9. WATCH PROBLEM

We know that at 3 o'clock, the hour hand and minute hand of a watch are exactly at right angles. At what exact time, will the two hands be at right angles again?

Hints on p. 72 Solution on p. 108 Comments on p. 198

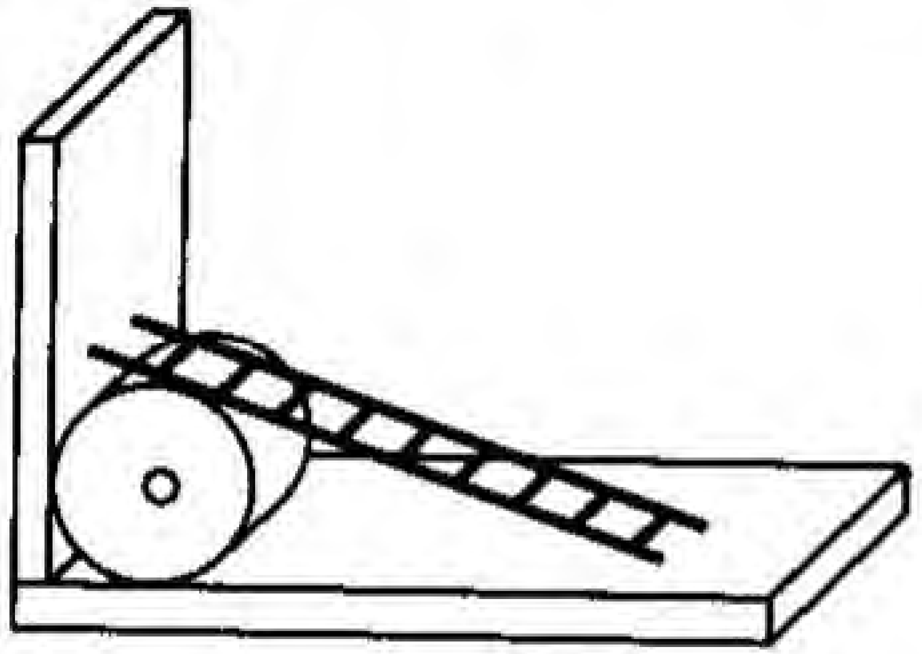
10. EQUALISE AMOUNTS

Three persons A, B and C have unequal amounts of money. A gives B half as much as B has and a rupee more. Then, B gives C half as much as C has and a rupee more. Lastly, C gives A half of what A then has and a rupee more. After this A, B and C all have equal amounts. Find the minimum amount of money each had in the beginning. All transactions are in integral number of rupees.

Hints on p. 73 Solution on p. 108 Comments on p. 198

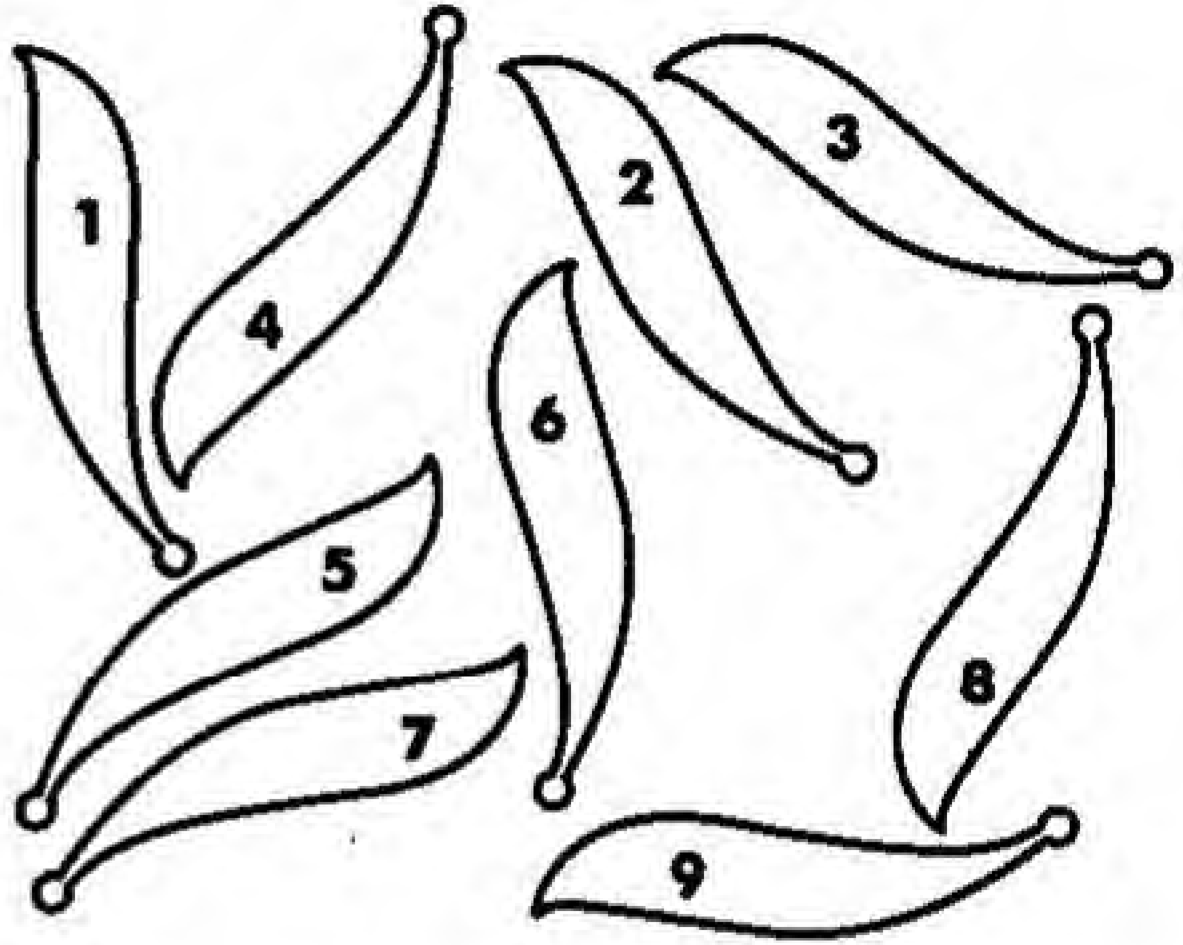
11. LADDER PROBLEM

A barrel of diameter 2 meters rests with its curved surface in contact with the ground and a wall. A 6.5 meters long ladder lies in contact with the barrel, the wall and the ground. What is the maximum distance between the ground end of the ladder and the wall?



Hints on p. 73 Solution on p. 110 Comments on p. 200

12. WRONG MAN OUT



All the nine figures are without exception congruent. But, in a certain respect one of them, differs from the other eight figures. Find the figure and tell how it differs?

Hints on p. 73 Solution on p. 111 Comments on p. 200

13. MISSING NUMBERS

In the following table, find the appropriate numbers that must be written in the four vacant cells.

	7	6	5	9
3	11		7	15
8		34	27	55
4	17	14	11	
6	29	24		39

Hints on p. 73 Solution on p. 111 Comments on p. 200

14. ALPHAMATICS

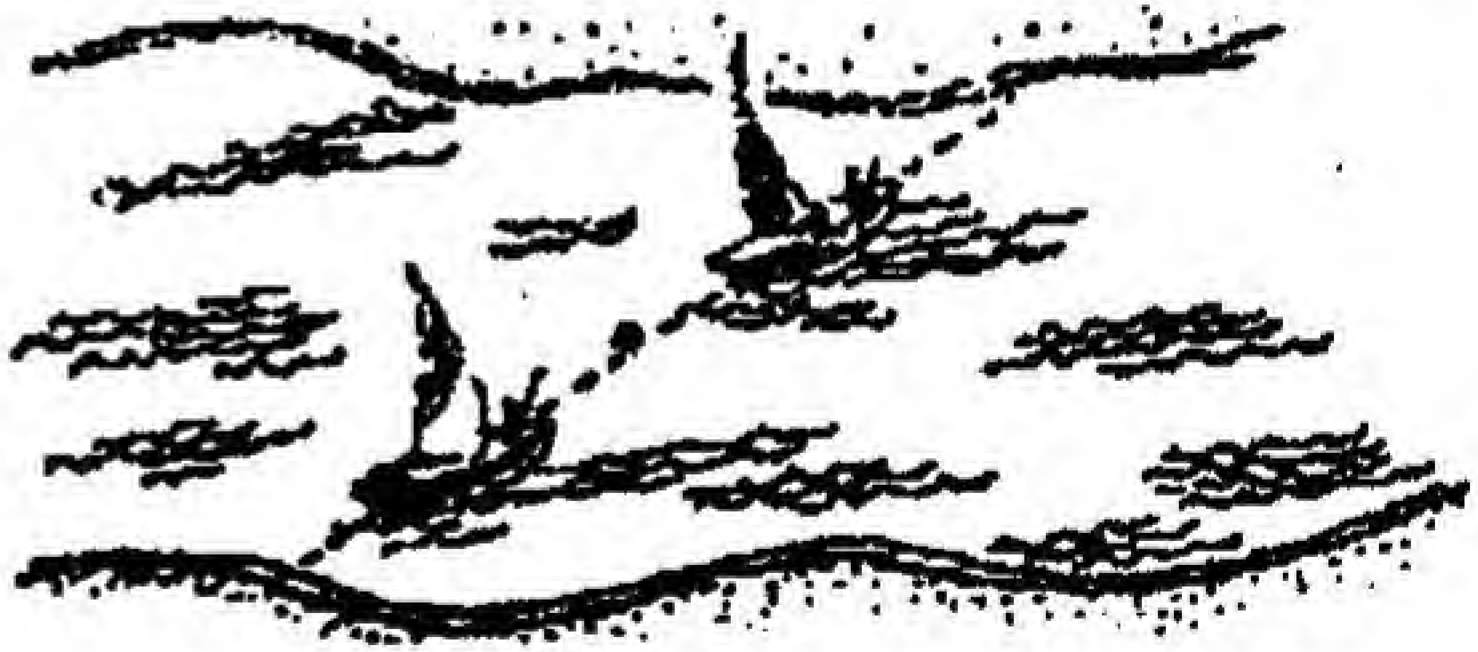
Replace the alphabets by different digits (except 4 and 9) so that the following addition sum may be correct

```
 GOOD
   TO
   BE
 ----
 TRUE
```

Hints on p. 74 Solution on p. 112

15. TWO BOATS

Two boats sail with uniform but different speeds in opposite directions between the banks of a river along the same straight line. After they cross each other at a point of trisection between the banks, they next cross each other midway between the banks. (Each boat on reaching the bank, took just two minutes to reverse.) Compare their speeds.



Hints on p. 74 Solution on p. 112 Comments on p. 201

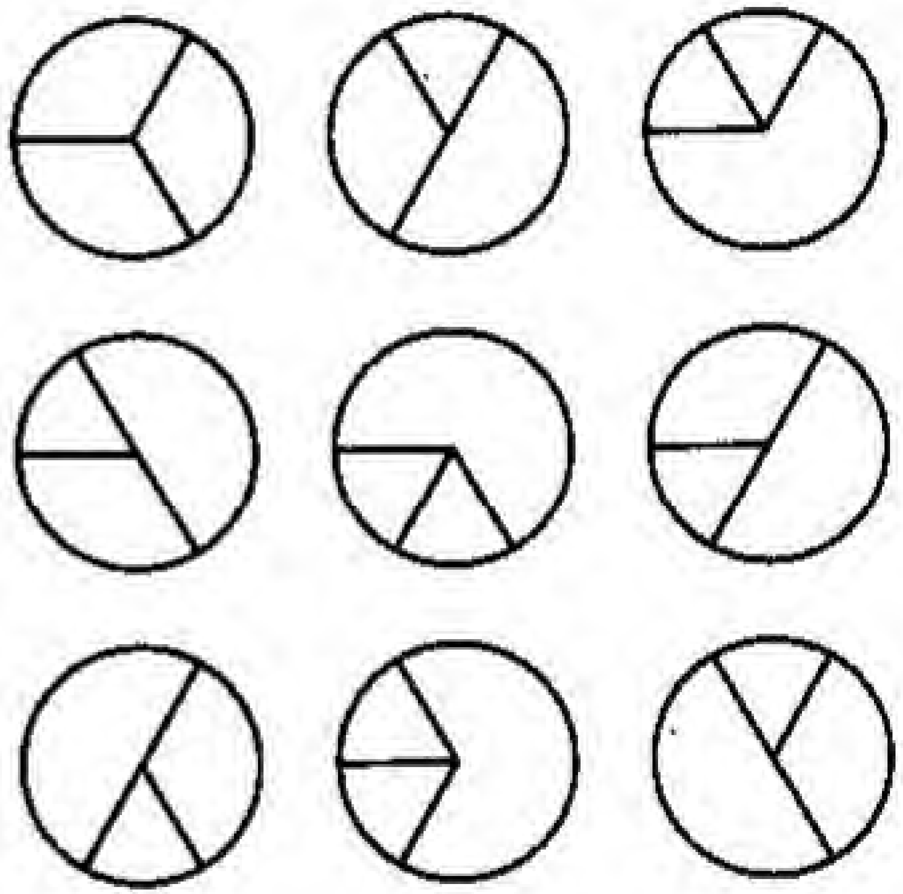
16. FOUR CHILDREN

Find the number of ways in which eight rupees can be distributed among four different children, so that every child gets at least a rupee but not more than three rupees.

Hints on p. 74 Solution on p. 113 Comments on p. 201

17. MISSING FIGURE

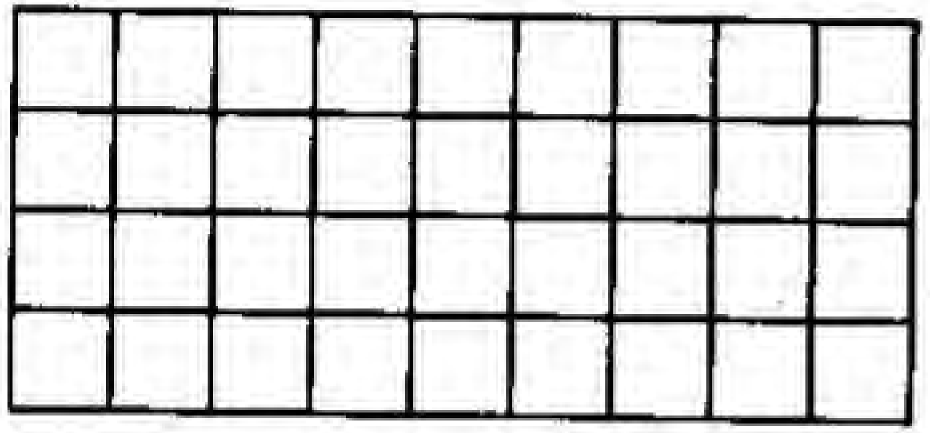
Out of a set of ten figures, nine are shown in the diagram below (in random order). Supply the missing figure.



Hints on p. 74 Solution on p. 114

18. DISSECTION PROBLEM

Cut up a rectangular region of length 9 units and width 4 units into two pieces which can be rearranged to form a square.



Hints on p.74 Solution on p. 115 Comments on p. 201

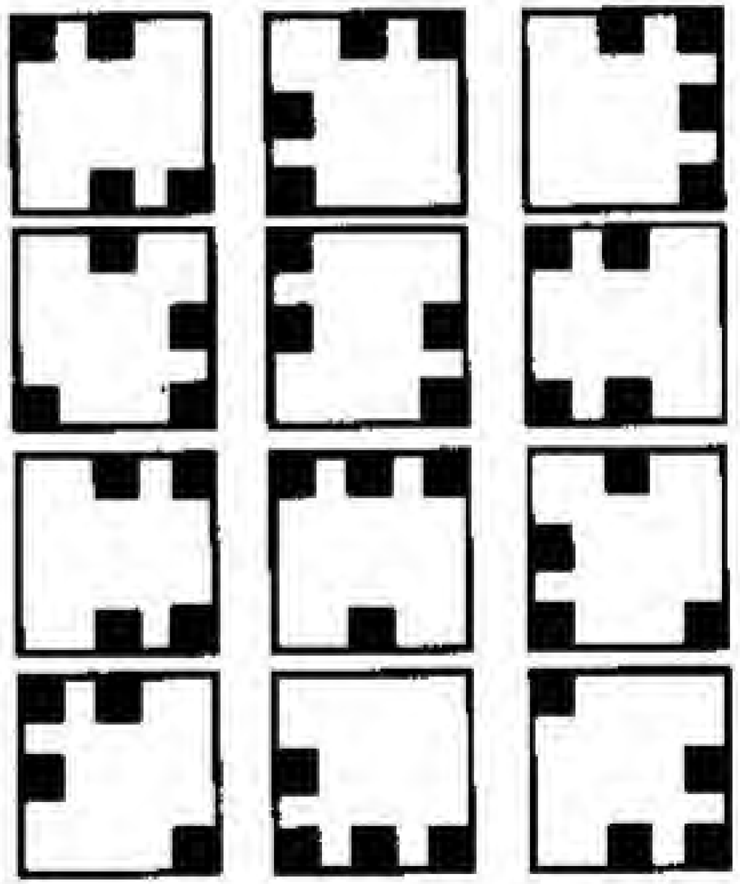
19. BLACK AND WHITE CAPS

Of the three prisoners A, B and C, it was surmised that C was the cleverest. To confirm this the warden carried out the following test. Summoning them to his room, he placed on his table five caps three black and two white in full view of the prisoners. Then, they were blindfolded and a black cap was placed on each prisoner's head. The warden then hid the two remaining white caps inside the drawers. After this, the blindfolds were removed and the warden asked A first whether he knew the colour of his cap. A, naturally replied "I don't know". Then B was asked if he knew the colour of his (B's) cap.

He also replied, "I don't know". But, when C was asked the same question, he said, "Yes, I know. I am wearing a black cap." On being asked to prove, C logically proved that he was having a black cap. Can you find out how C could logically prove the fact ?

Hints on p. 75 Solution on p. 116 Comments on p. 202

20. WHAT IS A 'PLUNKY'?



In the figures, all the four figures in the first column are "Plunkies". None of the four figures in the second column is a "Plunky". Which of the four figures in the third column are "Plunkies"?

Hints on p. 75 Solution on p. 116 Comments on p. 204

21. SPECIAL TRIPLET

If three persons are inter-related or if they are totally unrelated, they will be said to form a special triplet. In a set of any six persons, show that there is at least one special triplet.

Hints on p. 75 Solution on p. 117 Comments on p. 204

22. WHO ARE BROTHERS?

There are six men A, B, C, D, E, F some of whom always speak the truth and the rest always tell lies. Also, it is found that if any two of them are brothers, they belong to the same category and conversely. Now, A says that B and F are brothers. B says that D and E are not brothers. C claims to be the brother of F, D denies that he is the brother of A. E says that B and C are brothers and F says that A and E are brothers. Find out who is whose brother.

Hints on p. 75 Solution on p. 118

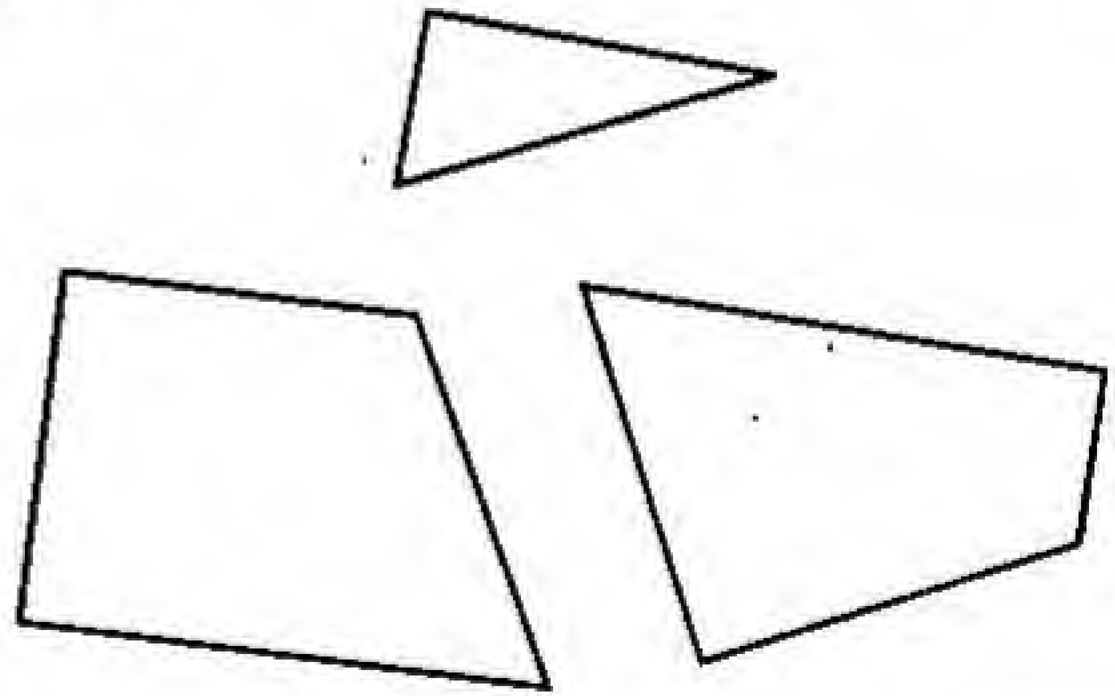
23. DIVISION BY 7

Explain why every six-digit number of the form ABABAB (for example 535353) is always divisible by 7.

Hints on p. 76 Solution on p. 118 Comments on p. 204

24. MAKE A SQUARE

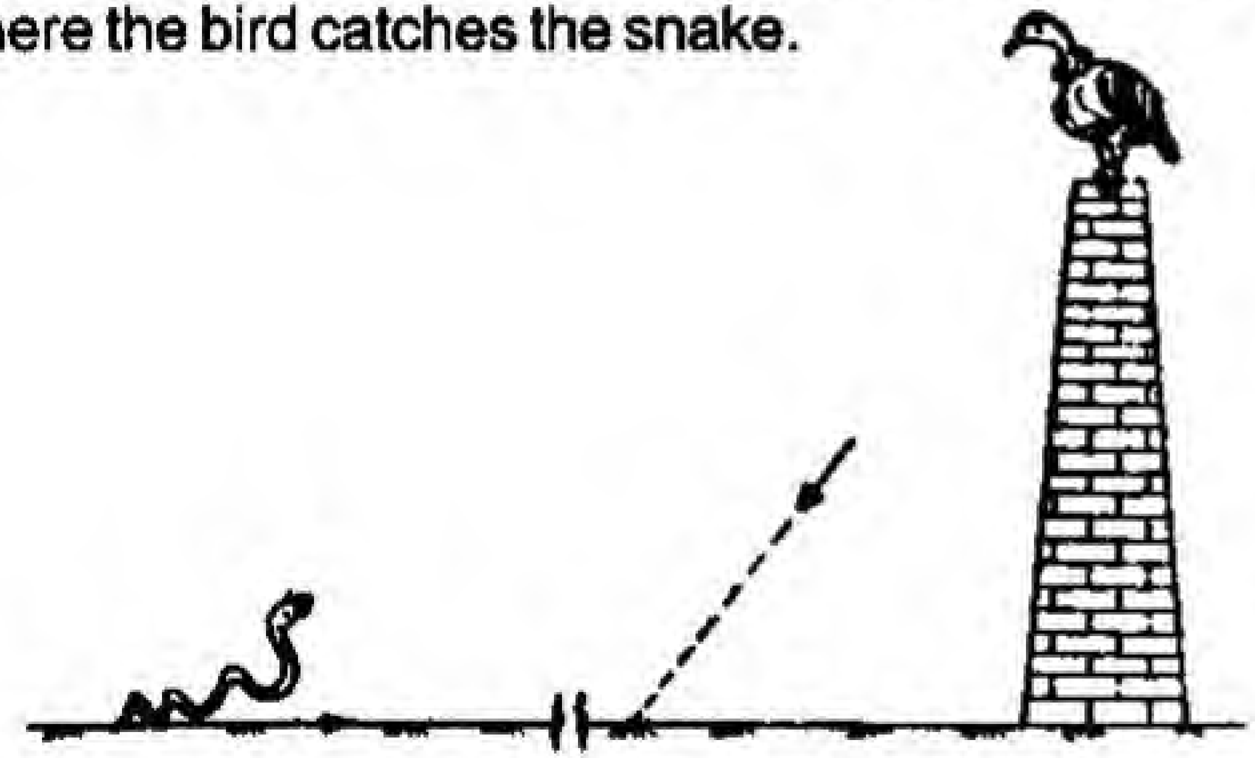
Using a thick paper cut out two sets of a triangle, a quadrilateral and a trapezium exactly of the size shown in the figure. Put the six pieces together, as in a jigsaw puzzle, to make a square.



Hints on p. 76 Solution on p. 119 Comments on p. 204

25. BIRD AND SNAKE

A vulture sitting on the top of a tower 20 meters high sees a snake on the ground 100 meters away from the foot of the tower. The vulture at once swoops down in a slant straight line to catch the snake, crawling at a uniform speed towards the tower. If the speed of the bird is the same as that of the snake, find where the bird catches the snake.



Hints on p. 76 Solution on p. 120 Comments on p. 205

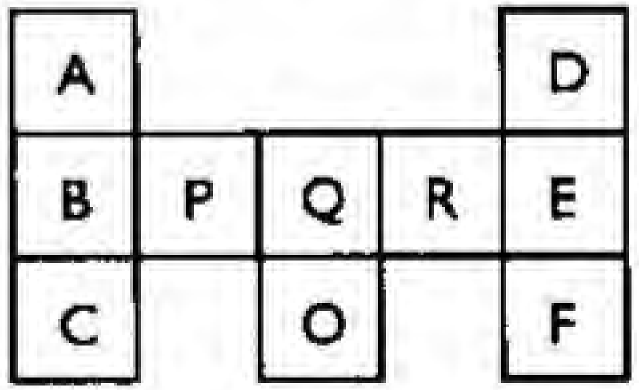
26. 12 BALLS PROBLEM

You are given 12 balls, all looking exactly alike. The weights of all the balls are perfectly equal except one ball whose weight very slightly differs from any of the rest. It is not known whether the slight difference in weight is by excess or defect. Using a balance only three times, show how to find the imperfect ball.

Hints on p. 77 Solution on p. 121 Comments on p. 205

27. PARKING PUZZLE

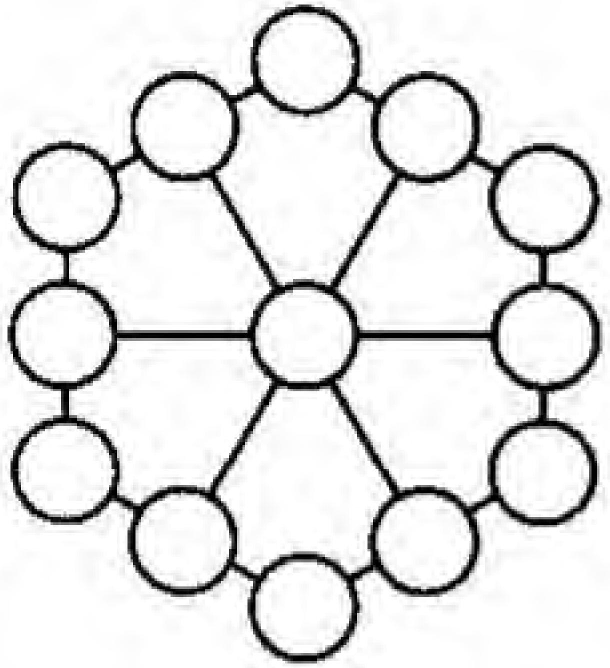
In the adjoining figure, place three one rupee coins (or black buttons) in the squares marked A,B,C and three five rupee coins (or white buttons)in the squares marked D,E,F. Move the coins (buttons) one by one, from square to square, so that in the end, the five rupee coins (white buttons) are on A,B,C and the one rupee coins (black buttons) are on D,E,F. Not more than one coin (button) can occupy a square at a time. Count the number of moves in your solution.



Hints on p. 78 Solution on p. 122 Comments on p. 206

28. MAGIC THIRTEEN

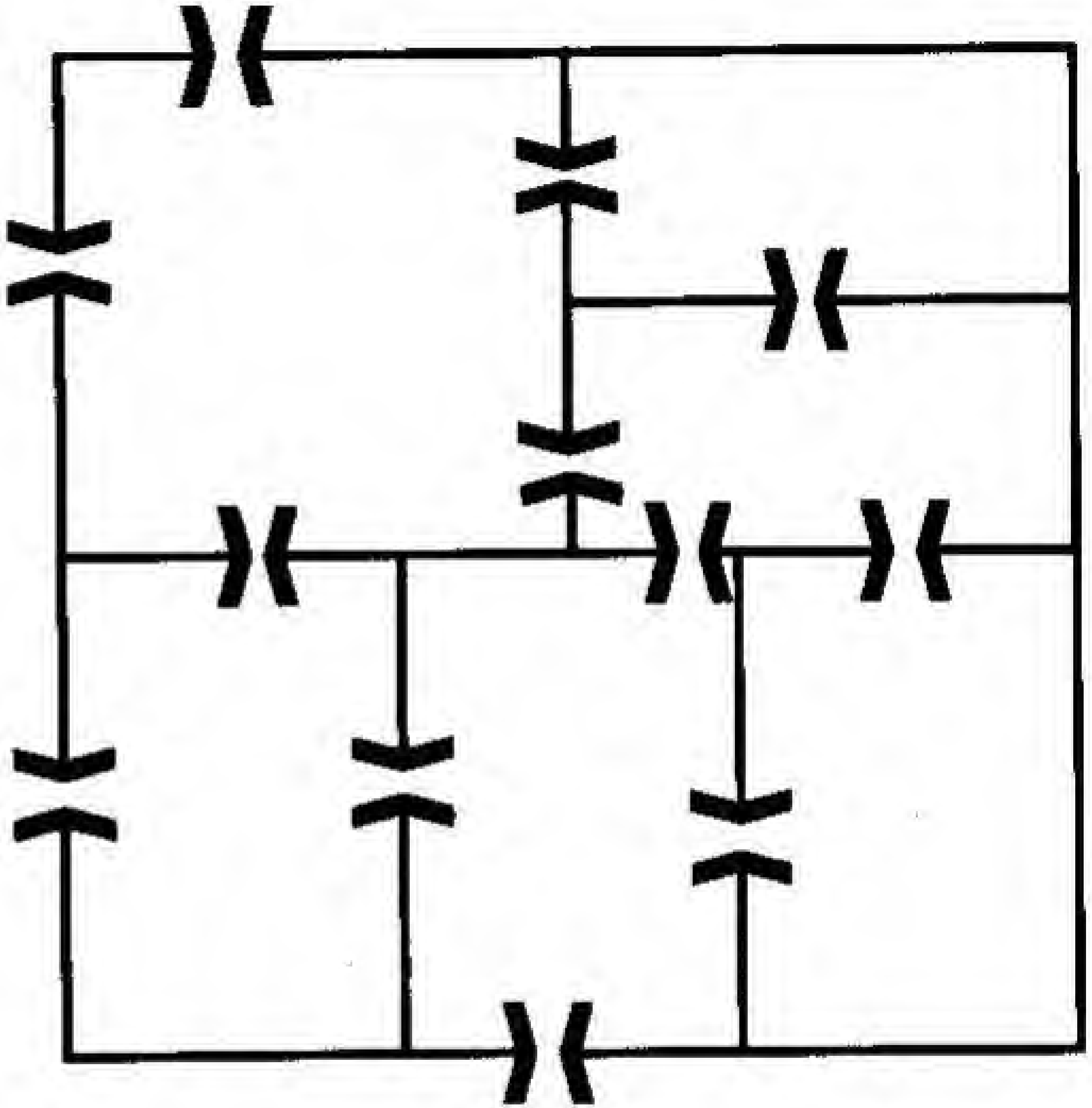
Insert the numbers 1 to 13 one in each of the circles in the given diagram so that the sum of the three numbers in every line is always the same.



Hints on p. 78 Solution on p. 122 Comments on p. 207

29. EXITS & ENTRANCES

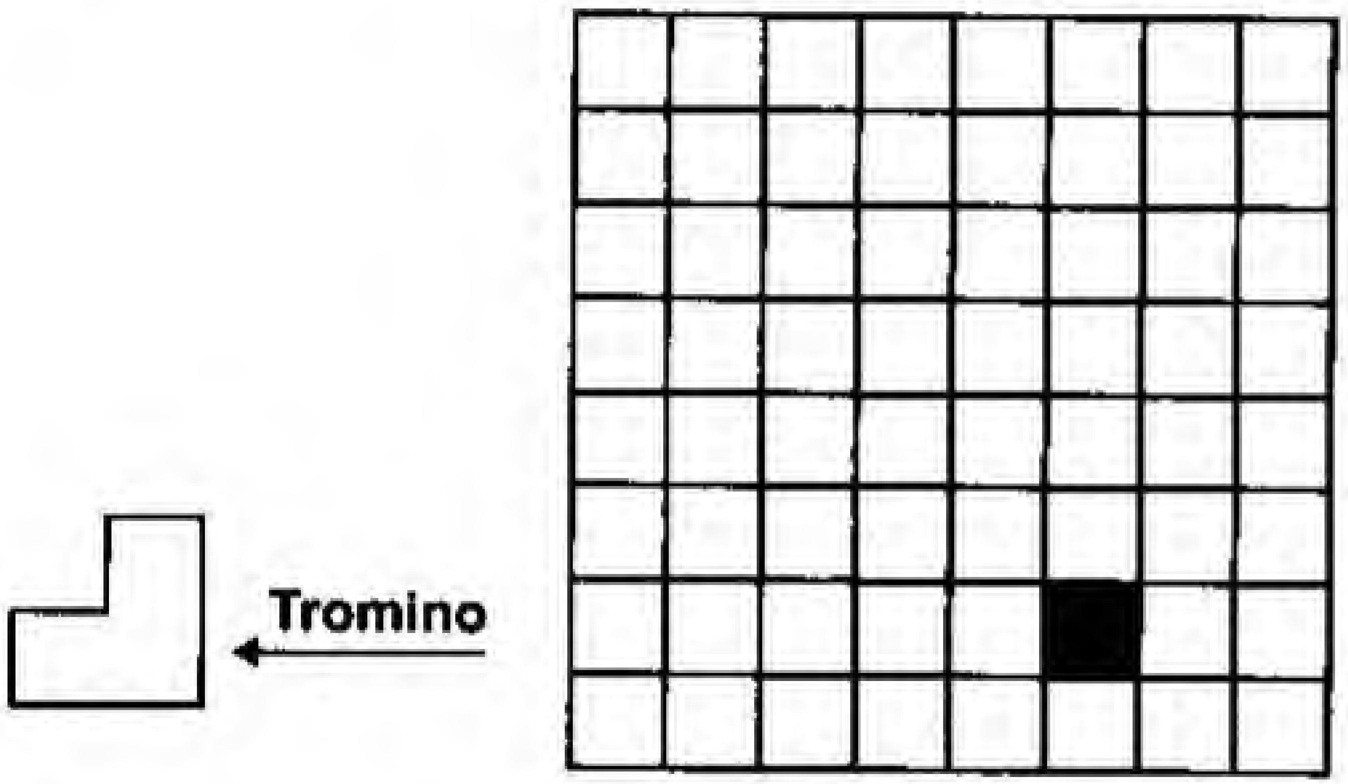
The diagram shows the plan of a house with a number of doorways in the walls of each room. The problem is to pass through each doorway but once only. You may start from outside or from inside of any room and end up anywhere.



Hints on p. 78 Solution on p. 124 Comments on p. 207

30. TROMINO PUZZLE

Three equal squares put together as in the figure given below constitute a 'tromino'. The problem is to accommodate 21 such trominoes in a 8 x 8 square board with one square deleted.



Hints on p. 78 Solution on p. 124 Comments on p. 208

31. PECULIAR NUMBERS

12	**16**	**20**	**30**
23	**37**	**51**	**65**
5	**10**	**17**	**50**

Examine the scheme of 12 numbers. It has 3 rows and in each row there are 4 numbers.

There is a property 'A' which is possessed by all the numbers in the first row, by none in the second row and by two in the third row.

There is a property 'B' which is possessed by all the numbers in the second row, by none in the third row and by two in the first row.

There is a property 'C' which is possessed by all the numbers in the third row, by none in the first row and by two in the second row.

What are the properties A, B and C?

Hints on p. 79 Solution on p. 125

32. STRANGE ALPHAMATICS

Replace the different letters by different digits so as to make the following multiplication sum true and show that the solution is unique.

TEN × TEN = DOZEN

Hints on p. 79 Solution on p. 125

33. DEVOUT PRIEST

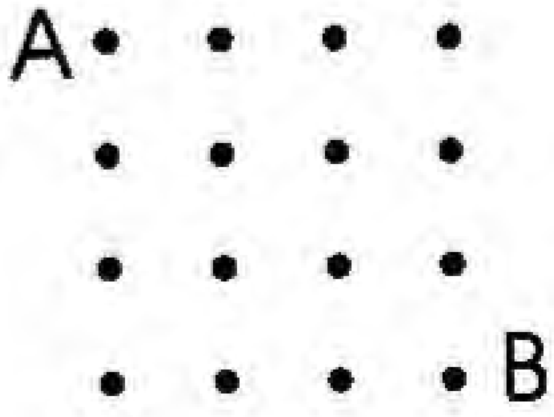
There were 3 temples and each temple had a pond in front of it. The water in the ponds had the magical property - if a basket containing some flowers was dipped into the water, the number of flowers would increase by 50%.

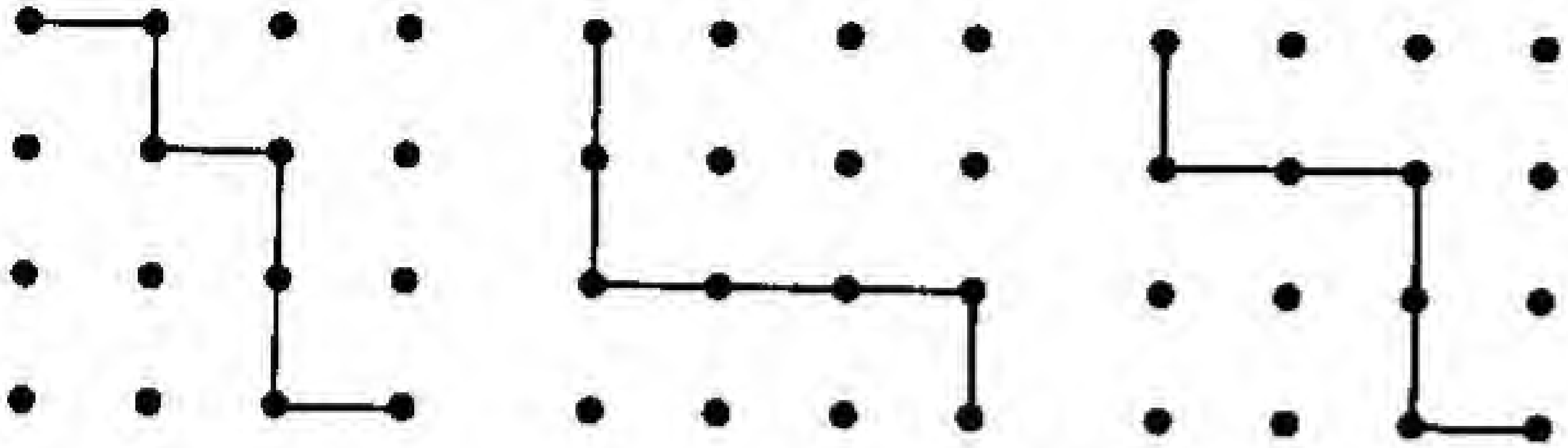
A devout priest went to all the three temples every day and made an offering of same number of flowers to each temple. Before offering the flowers at any temple the priest would first dip the flower basket in the pond in front of that temple. After making an offering in the third temple, he would go home with an empty basket. Find how many least number of flowers he carried in his basket before he dipped it in the first pond. How many flowers did he offer in each temple?

Hints on p. 79 Solution on p. 126 Comments on p. 209

34. EAST-SOUTH ROUTES

There are 16 points in a square formation with four rows with four points in each row. You are required to go from the corner point A to the corner point B. You can move from point to point eastwards or southwards only. As specimen, three possible routes have been shown below. Actually, there are many different ways of doing this. The question is, how many?





Hints on p. 80 Solution on p. 127 Comments on p. 211

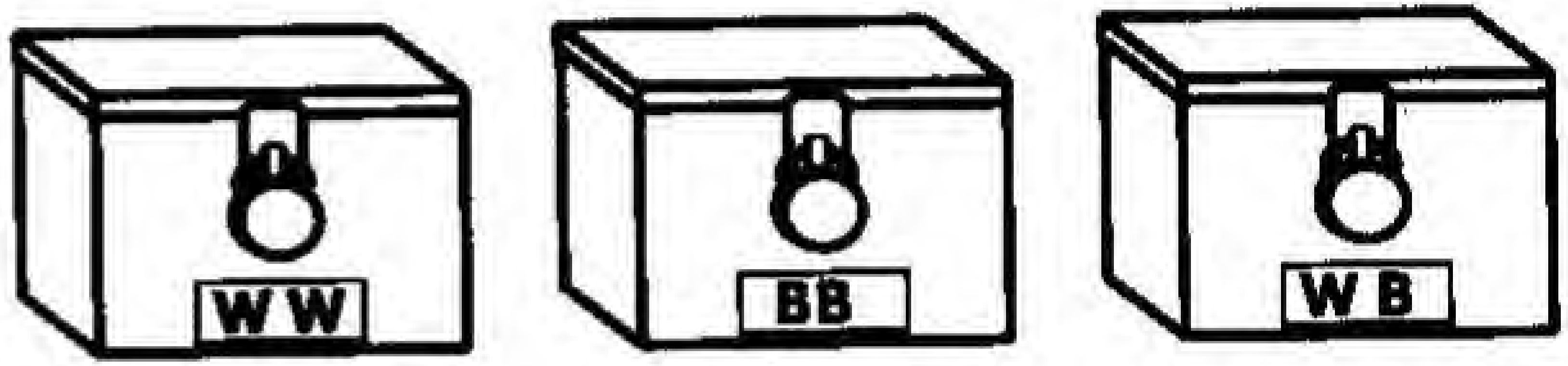
35. AN UNKNOWN DIGIT

All the digits, except the 26th digit in a number N of 50 digits are 1's. If N is divisible by 13, find the 26th digit.

Hints on p. 80 Solution on p. 129 Comments on p. 213

36. WRONG LABELS

A closed box contains two white balls, another closed box two black balls and a third one a white and a black ball. It is not known which box contains what balls. Three labels with letters WW, BB and WB are pasted one on each box, but all wrongly, W standing for white and B for black. You are allowed to draw blindfolded some balls from the boxes and then see the colour of the balls drawn. What is the least number of balls you will draw which will enable you to discover the contents of each box ?



Hints on p. 80 Solution on p. 129 Comments on p. 214

37. DOUBLE AMOUNT

Arun tells Ashok, "If you give me 9 rupees, I will have twice as much as you will have". "But", tells Ashok to Arun, "If you give me only 5 rupees, I will have twice what you will have". How much money did each have ?

Hints on p. 80 Solution on p. 130 Comments on p. 215

38. PILGRIM SADHU

On a fine Monday morning at exactly 8 o'clock, a sadhu started climbing the sacred Mount Girnar and he ascended with varying speeds and halted at some places for quenching thirst. He reached the Ambaji temple on Mount Girnar exactly at 2.00 p.m. on the same day. Here, he stayed for about five days. On Saturday, he left the temple exactly at 8.00 a.m. and descended with varying speeds. He reached the foot of the hill at exactly 2.00 p.m. on the same day. Show that there is just one point of the route where the sadhu was at the same time (between 8.00 a.m. and 2.00 p.m.) both while he was ascending and while descending.

Hints on p. 80 Solution on p. 130

39. FARMER'S WILL

A farmer wanted to distribute his property among his three sons. A part of his property consisted of 17 cows. He wrote in his will that half of this should go to his first son, one-third to his second son and one-ninth to his third son. After the death of the farmer, his sons found it impossible to partition the cows according to the will. They approached a judge and requested him to find a solution. The judge did find a solution. How?

Hints on p. 81 Solution on p. 131 Comments on p. 216

40. A DICE TRICK

Ask a friend to place two dice one over the other on a table, when you are out of sight and cover the top face of the upper dice with some flat object like a postcard. Then you come back, examine the visible sides of the dice and announce the number

of dots on each of the four hidden sides. How can you do this?

Hints on p. 81 Solution on p. 131 Comments on p. 217

41. DIGITS IN A TRIANGLE

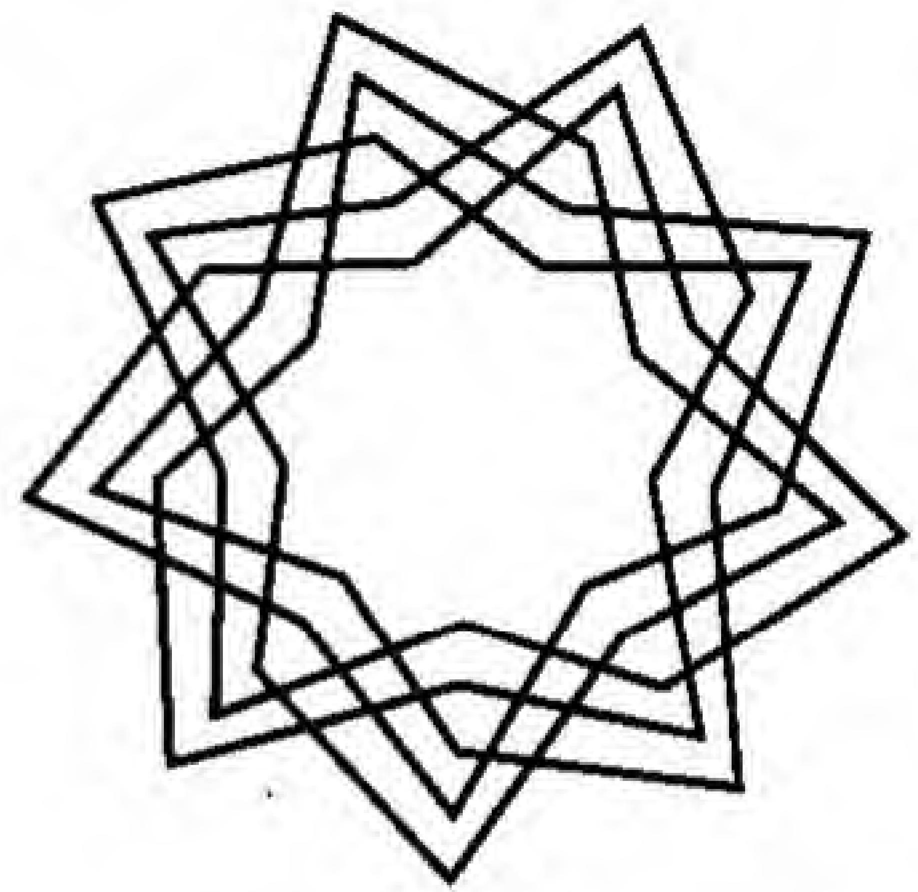
3 8 1 0 4 9	3 6 5 0 4 6
5 7 1 4 5	1 2 1 4 4
2 6 3 1	5 0 0 2
4 3 2	1 0 2
1 1	3 2
0	4

Here are two schemes of digits each written in a triangular formation. Can you find some pattern in each of them? These are two different independent problems. It is easier to find a pattern in the first (left side) scheme than the one on the right.

Hints on p. 82 Solution on p. 132 Comments on p. 218

42. UNICURSAL DESIGN

The design (rangoli) on the next page is unicursal or single circuited, that is, it can be drawn without lifting the pencil off the paper or retracing any line. Take a piece of blank paper and pencil and try to draw this design (free hand) without using a tracing paper.



Hints on p. 82 Solution on p. 133 Comments on p. 218

43. AN AGE PROBLEM

Maya's husband is only three years older than her. Dora, their daughter was born five years after their marriage. Maya says to Dora, "How wonderful! 17 years ago, I was as old as you

will be 17 years hence". How old was Dora's father at the time of his marriage?

Hints on p. 82 Solution on p. 134 Comments on p. 220

44. REVERSED DIGITS

A five-digit number, when multiplied by 4 yields the number with all its digits reversed. All the digits in the number are different. Find the number.

Hints on p. 82 Solution on p. 134 Comments on p. 220

45. PENTAGRAM NUMBERS

Write down each of the ten numbers 1 to 12 twice, dropping 7 and 11. Arrange these twenty numbers in five rows with four numbers in each row, in such a way that the following conditions are satisfied.

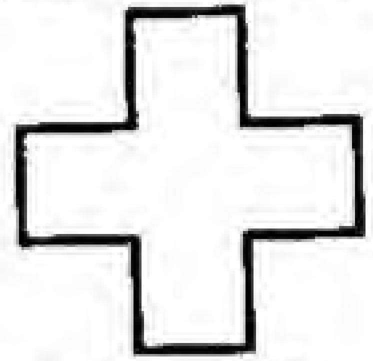
1.The four numbers of every row sum up to the same constant.

2.There is one and only one number common to every pair of rows.

Hints on p. 83 Solution on p. 135 Comments on p. 221

46. CROSS AND SQUARE

From a piece of thick paper cut out the given figure of a cross. Divide this figure into five pieces by straight cuts in such a way that these pieces can be reassembled to form a square.



Hints on p. 83 Solution on p. 136 Comments on p. 222

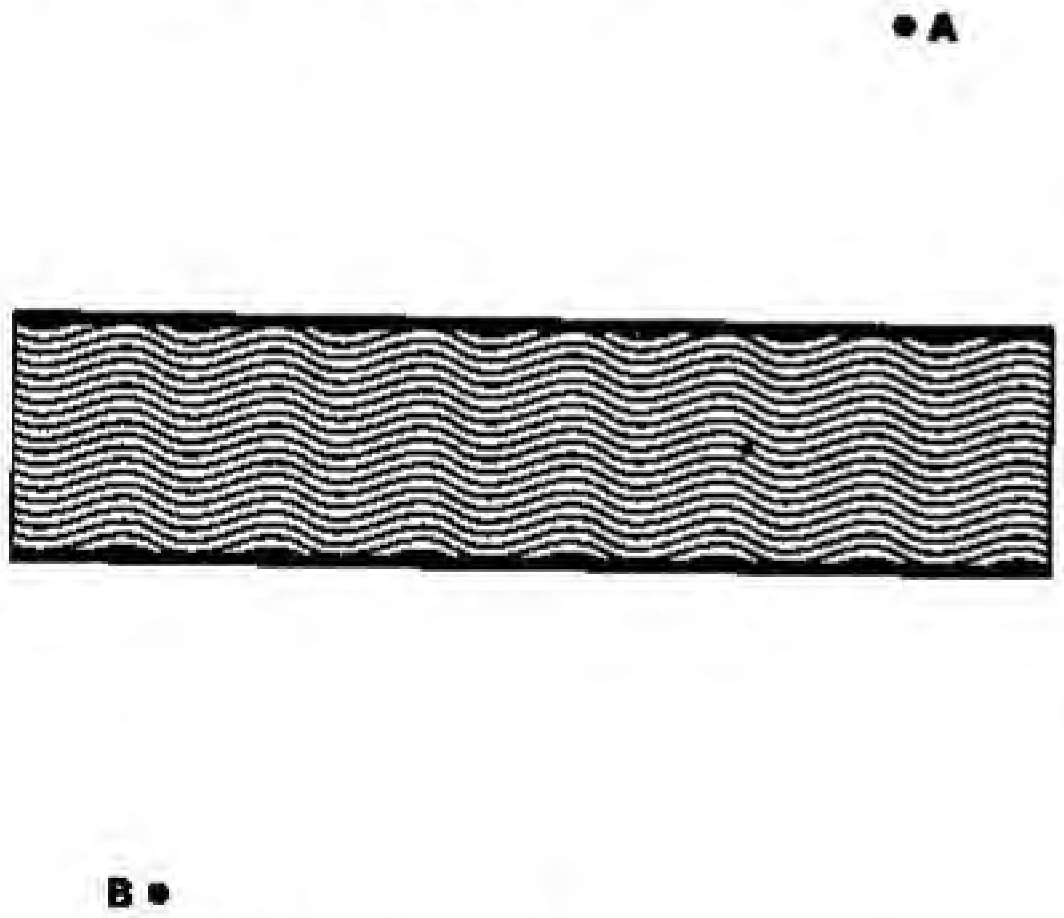
47. A COINS PROBLEM

A merchant had a total of 100 coins in his cash box, the coins being in 10 paise, 50 paise, 1 rupee and 5 rupee denominations, and their total value was thirty five rupees. When a teacher, who was his customer, came to him with six rupees for change, so that he might give 10 paise to each of his sixty pupils, the merchant said, "I'm sorry, I have only Rs. 5-00 worth of 10 paise coins". How many coins did the merchant have in each denomination?

Hints on p. 83 Solution on p. 136

48. A BRIDGE PROBLEM

Two friends live in houses A and B on the opposite sides of a river, some distance away from the banks. Where should a bridge be built on the river so that one can go from A to B using the bridge to cross the river and by the total shortest route. Assume that the two banks of the river are straight and parallel. Also the bridge has to be perpendicular to the banks.



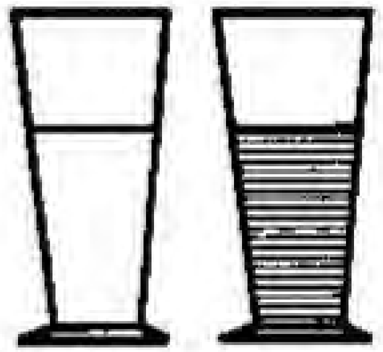
Hints on p. 84 Solution on p. 137 Comments on p. 222

49. THE MISSING DIGIT

Ask your friend to do the following without your knowledge. "Write down any number of four or five digits. Subtract from it the sum of its digits. Strike out any one digit in the answer and tell me the remaining digits". As soon as he tells you those digits, you tell him the digit that he struck out. How do you tell him this digit?

Hints on p. 84 Solution on p. 138 Comments on p. 223

50. WATER AND WINE

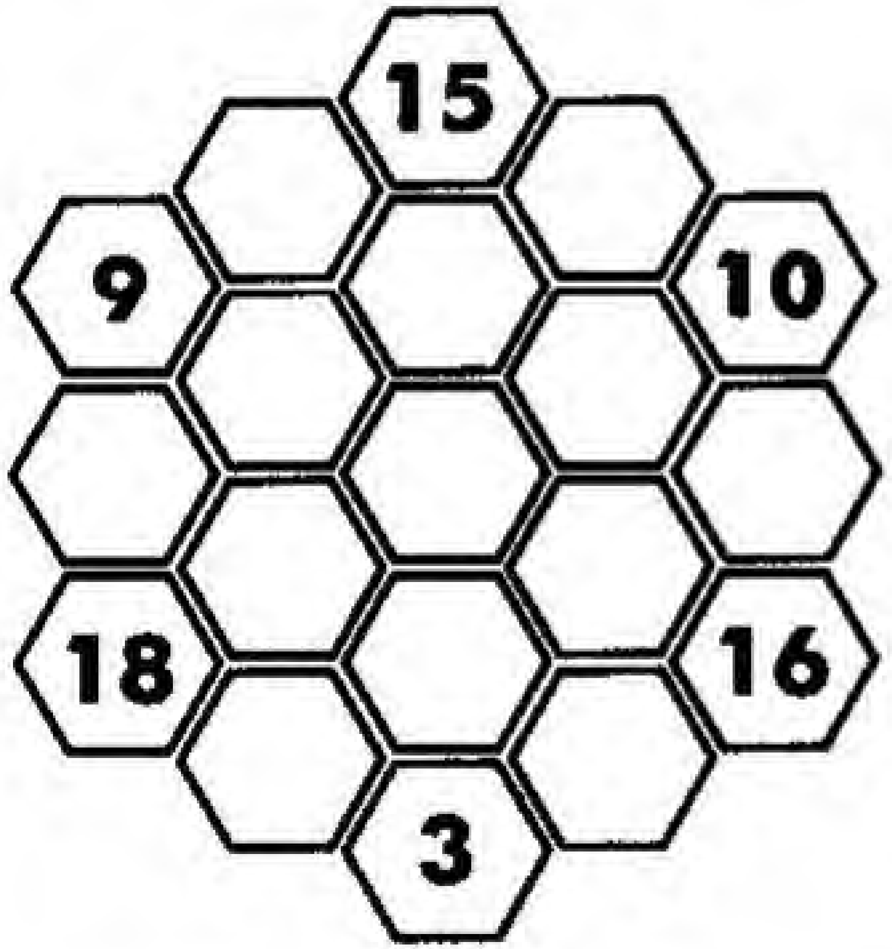


A glass is half full of water and another exactly similar glass is half full of wine. A spoonful of wine from the second glass is added to the first glass and stirred. Then a spoonful of the mixture from the first glass is added to the second glass. Is the net quantity of wine removed from the second glass more or less than the net quantity of water removed from the first glass?

Hints on p. 84 Solution on p. 138 Comments on p. 224

51. MAGIC HEXAGON

A hexagonal diagram with nineteen small hexagonal cells is given. Write numbers from 1 to 19, one in each cell so that the sum of the numbers in each line is the same. Six of the numbers are already given. You are required to fill up the remaining thirteen empty cells.



Hints on p. 84 Solution on p. 139 Comments on p. 224

52. REMAINING REMAINDER

"Take any three digits in a cyclical order. Take the three two digit numbers formed from them in the same cyclical order. Divide these three two-digit numbers by 13 and tell me two of the remainders in the proper order in which they occur and I will tell you the third remainder". This is what you tell your friend. But, how will you tell him the third remainder? Let us explain the problem by an example. Suppose your friend takes the three digits 9,6,4 or 6,4,9 or 4,9,6, which are all cyclically, equivalent. The three two-digit numbers your friend will have are 96, 64, 49 in this or any cyclically equivalent order. Dividing these by 13, he will get the remainders 5, 12, 10 in this order. Then, if he gives you the numbers 5 and 12 (not 12 and 5), can you tell him that the third remainder is 10? Or from 12 and 10 (not 10 and 12) can you find 5 or from 10 and 5, can you find 12 ?

Hints on p. 85 Solution on p. 140 Comments on p. 225

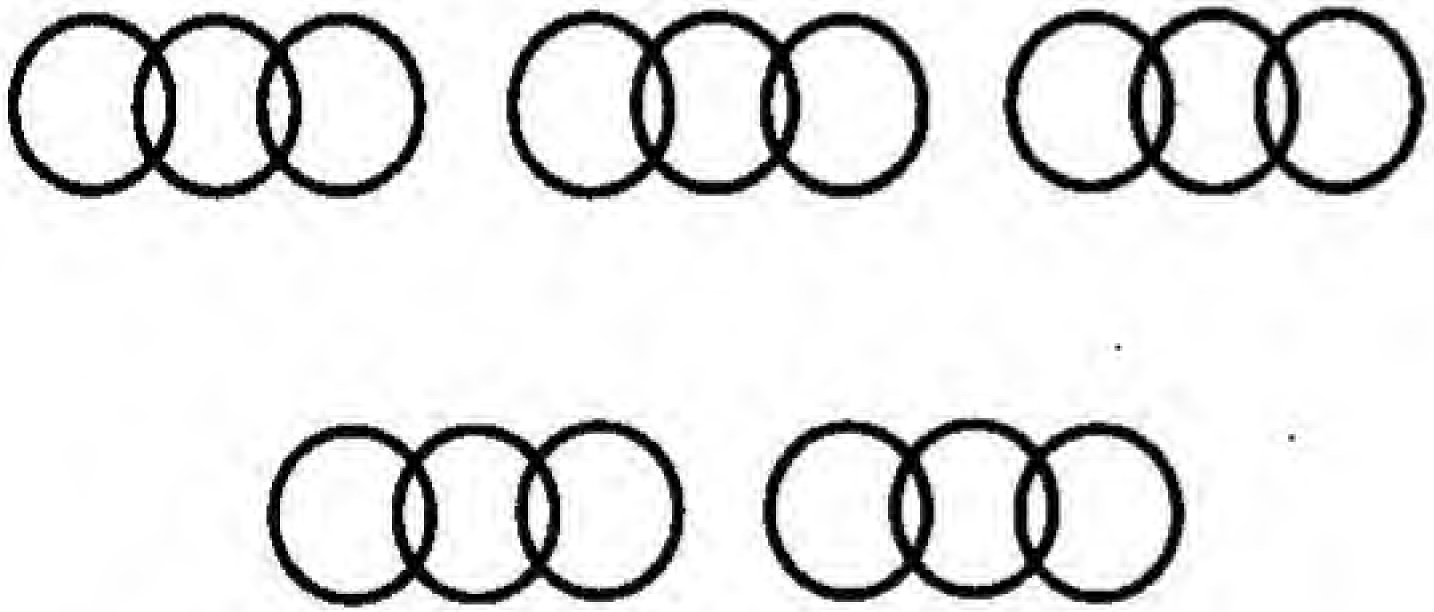
53. A MULTIPLE OF 11

Using nine different digits, write down the biggest multiple of 11.

Hints on p. 85 Solution on p. 141 Comments on p. 226

54. A CHAIN PROBLEM

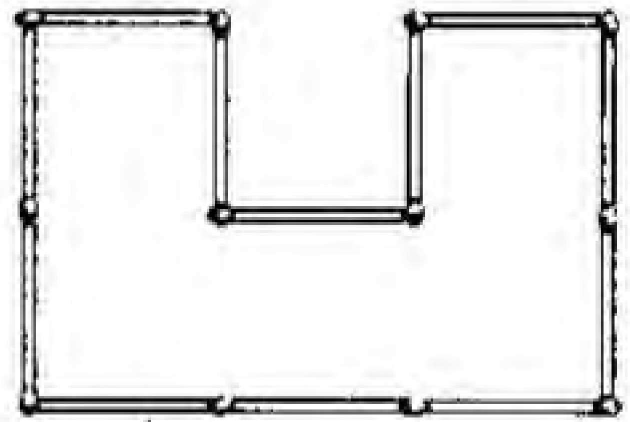
A rich man had five bits of gold chains each with three links. He went to a goldsmith to have them made into one long chain. The goldsmith said that the cost of breaking open one link is 1 rupee and closing it by soldering is 2 rupees. What is the least amount the man had to spend to make one long chain out of the bits?



Hints on p. 85 Solution on p. 142

55. TWELVE MATCHES

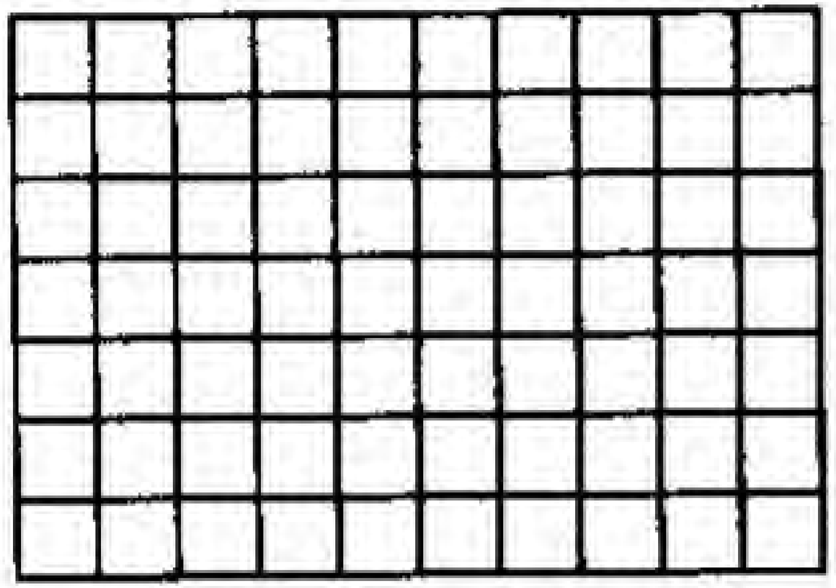
Twelve matches of the same length enclose a space as shown in the figure. The area of this space is five 'match squares'. Rearrange the matches so that they enclose an area equal to four 'match squares'.



Hints on p. 86 Solution on p. 142

56. HOW MANY SQUARES ?

How many total number of squares, big and small, are there in this figure?



Hints on p. 86 Solution on p. 143 Comments on p. 226

57. WEDDING INVITEES

120 invitees came for a rich man's wedding reception. Many had come in 50 vehicles parked in the compound of the rich man's bungalow. A boy counted the wheels of all these vehicles and announced that there were 139 wheels. Each car carried five visitors, each auto-rickshaw a couple and single persons came on bicycles. If 11 persons had walked to the reception, how many auto rickshaws were parked there?

Hints on p. 86 Solution on p. 143

58. BUYING COCOANUTS

The quantity of kernel inside a cocoanut is proportional to the volume of the cocoanut. A small cocoanut costs Rs. 5.00. But a big cocoanut whose diameter is one and half times that of a small one costs Rs.10.00. Which is more profitable to buy, a big cocoanut or a small one?

Hints on p. 86 Solution on p. 144

59. PROFIT AND LOSS

I bought two bicycles at different prices one for myself and another for my brother. After some time, we decided to buy a scooter and so sold the bicycles. Both bicycles fetched us the same price of Rs.1584. But, there was a profit of 10% on one bicycle and a loss of 10% on the other. Was there an overall profit, loss or neither?

Hints on p. 86 Solution on p. 144 Comments on p.228

60. A LIE

Shantilal was a nice gentleman and never believed in gambling with his bridge friends at the club. They played cards with marbles as stakes. One day, Shantilal won all rubbers, a good many of them, winning one marble in the first

rubber, three in the second, five in the third, and so on in successive odd numbers. In the end, he carried home a heavy carton load of marbles saying, "I will have a good present for my children today". Reaching home, he kept half of the marbles in a locked cupboard and left the other half on a tray on the table.

Returning home after a stroll, he found only one marble in the tray and the others had disappeared. "Ashok !" he called his son and asked him sternly, "Did you remove the marbles from this tray?" "Yes, Papa" confessed Ashok nervously. "How many did you take?" asked Shantilal. "Sorry, Papa, I don't remember. Asha also took some." "Asha !" shouted Shantilal. "Come here. How many marbles did you remove from here?" "I too don't remember, Papa" replied Asha apologetically. "If Ashok took some marbles, I thought I would also take some. But I don't know how many I took. The only thing I remember is that I took exactly half the number Ashok took." "Never mind" said Shantilal. "Any way, I was going to give the marbles to you only. You should have asked me. Did any one else come into the room?" "None" replied Asha.

After a little thought Shantilal concluded that Asha must have told a lie. How did Shantilal discover this?

Hints on p. 87 Solution on p. 145

61. HALF-DISTANCE - HALF TIME

Raju can go from home to school, running half the distance and walking the remaining half OR running half the time and walking half the time. Which of the two alternatives should he choose to reach school quicker?

Hints on p. 87 Solution on p. 145 Comments on p. 229

62. PERFECT SQUARE

Find the largest perfect square of five digits in base seven.

Hints on p. 87 Solution on p. 146 Comments on p. 229

63. RECTANGLE-SQUARE

Can you cut a rectangular sheet of paper into three pieces that can be rearranged, without overlapping, to form a square?

Hints on p. 87 Solution on p. 147 Comments on p. 230

64. RULER CONSTRUCTION

Given a line segment AB and its middle point M, using only an ungraduated ruler, find the points of trisection of the segment AB.

Hints on p. 88 Solution on p. 148 Comments on p.231

65. COLOUR CUBES

In how many ways can the six faces of a cube be painted with six given different colours?

Hints on p. 88 Solution on p. 149 Comments on p. 232

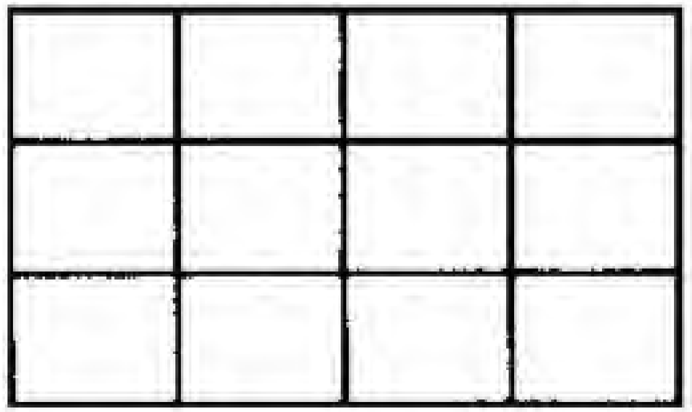
66. MAGIC OCTAHEDRON

Write numbers 1 to 8, one on each of the eight faces of a regular octahedron so that the sum of the numbers on the four faces meeting at every vertex is the same.

Hints on p. 88 Solution on p. 150 Comments on p. 232

67. RECTANGULAR MESH

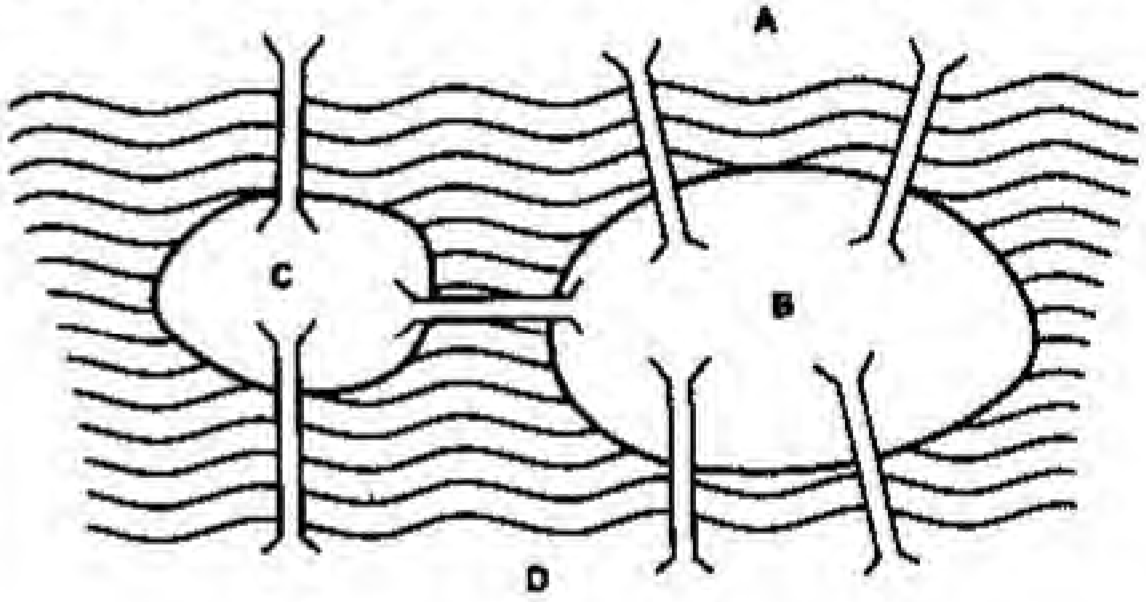
If it is required to draw this diagram in a continuous manner without retracing or crossing a line already drawn, it will be necessary to lift the pencil off the paper a certain number of times. What is the least number of times you will have to do this?



Hints on p. 88 Solution on p. 152 Comments on p. 233

68. KÖNIGSBERG BRIDGES

In the Pregel River in Königsberg (Germany) there are two islands which are connected by a bridge and they are connected with the mainland by six other bridges, as shown in the figure. Can you show a way by which one can walk over each of the seven bridges once only, starting from any point on land and ending at the same or any other point?



Hints on p. 89 Solution on p. 152 Comments on p. 233

69. A PARADOX

1. There are three false statements in this list.
2. Apple is a fruit.
3. Bombay is to the south of Ahmedabad.

4. There are ten letters in the word MATHEMATICS.
5. If $2x+5=17$, then $x=6$.
6. If two lines in space are not parallel, they must meet in a point.
7. Lion is a wild animal.

In the above list of seven statements, is the first statement true?

Hints on p. 89 Solution on p. 153

70. DIGIT PAIRS

3 1 2 1 3 2

4 1 3 1 2 4 3 2

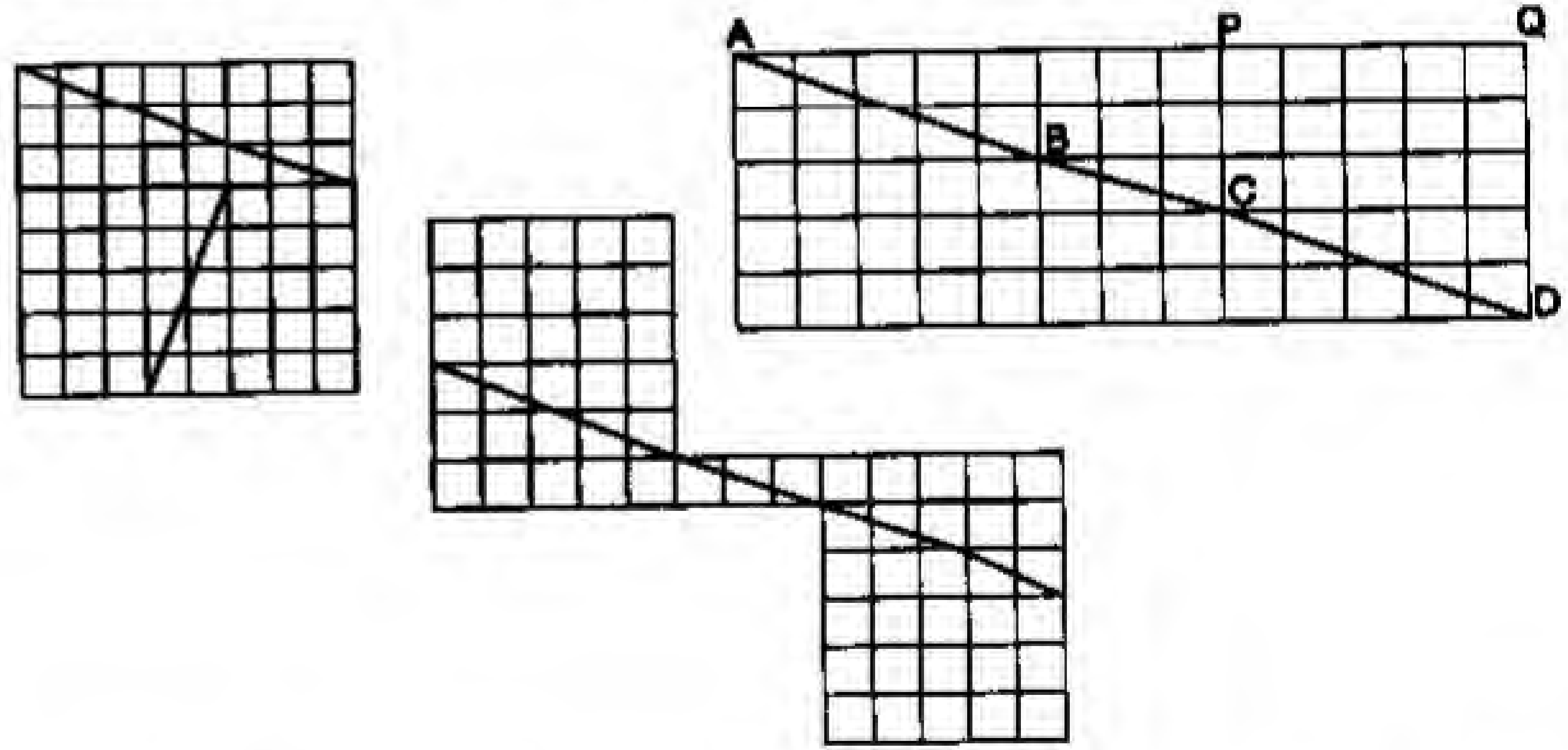
Note that in the first sequence of digits there is one digit between the two 1's, two digits between the two 2's and three digits between the two 3's. In the second sequence the same holds true and in addition, there are four digits between the two 4's.

Can you write down ten digits, consisting of pairs of 1's, 2's, 3's, 4's and 5's having the same property, but now, in addition, there must be five digits between the two 5's?

Hints on p. 89 Solution on p. 154 Comments on p. 234

71. 63 = 64 = 65.

From a piece of thick paper cut out a square of 8 units by 8 units. Draw parallel lines both ways to form a mesh of 64 small squares. Cut the square into four pieces along the thick lines exactly as shown in the first figure. Rearrange the pieces as shown in the second figure. Now, count the number of small squares. Are there 65? From where has the extra square come? Again, rearrange the same four pieces as shown in the third figure and count the small squares. Are there 63? How and where has one square disappeared?



Hints on p. 89 Solution on p. 154 Comments on p. 235

72. CONSTANT SUM

12	9	21	15	18
8	5	17	11	14
23	20	32	26	29
4	1	13	7	10
19	16	28	22	25

You see here twenty five numbers written in a square formation. Choose any five of these so that no two of the chosen numbers are in the same row or column. (This can be done in 120 different ways). Add the chosen numbers. Explain why you always get the same answer, 81.

Hints on p. 89 Solution on p. 156 Comments on p. 235

73. SQUARE HOLE IN A SQUARE

From a piece of thick paper, cut out a square of any convenient size. On this draw two lines perpendicular to each other, so that each line cuts a pair of opposite sides of

the square. The two lines should not be perpendicular to the sides of the square. Cut along these lines to get four unequal quadrilateral pieces. Now arrange these pieces to make a bigger square with a square hole somewhere inside..

Hints on p. 90 Solution on p. 157 Comments on p. 236

74. FOUR 4's

Many numbers can be expressed by using only the digit 4, four times along with arithmetical signs. We give below examples of such expressions for the number 7, 10, 21, 96.

$$7 = \frac{44}{4} - 4 \qquad 21 = \frac{\sqrt{4}}{.4} + 4 \times 4$$

$$10 = \frac{4 \times 4 + 4}{\sqrt{4}} \qquad 96 = \left(\frac{4}{.4}\right)^{\sqrt{4}} - 4$$

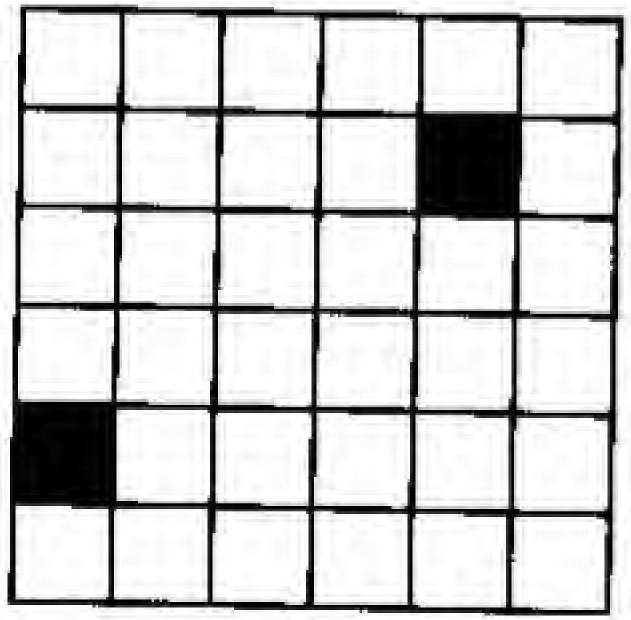
Find expressions for the numbers 13, 17, 52 and 68 by using only four 4's and arithmetical signs.

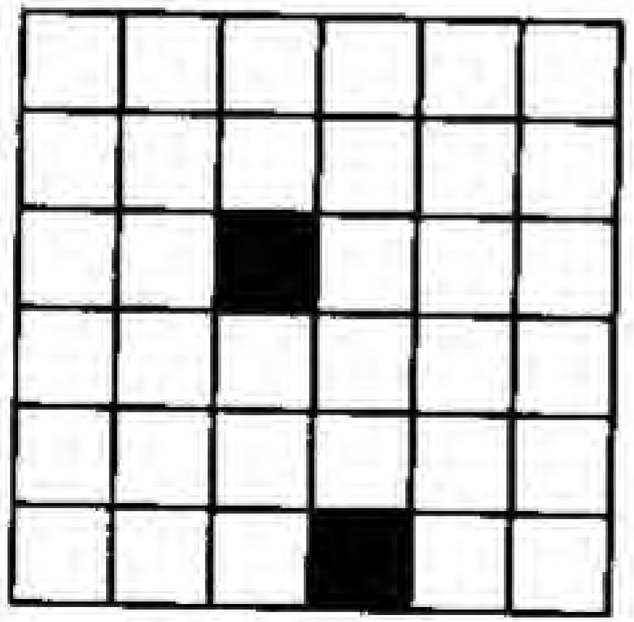
Hints on p. 90 Solution on p. 157 Comments on p. 237

75. SEVENTEEN DOMINOES

For this and the next problem, the reader will do well to make a few dominoes out of thick paper or cardboard. A domino is a rectangle, which is two units long and one unit wide (or two unit squares connected along a common edge). You will require 17 dominoes for this problem.

On another sheet of thick paper, prepare two 36-square meshes, 6 units by 6 units each, and two of the squares as shown in each figure below should be blocked out (i.e., to be considered as deleted). The problem is to accommodate the 17 dominoes in the remaining 34 squares in each. These are two problems.

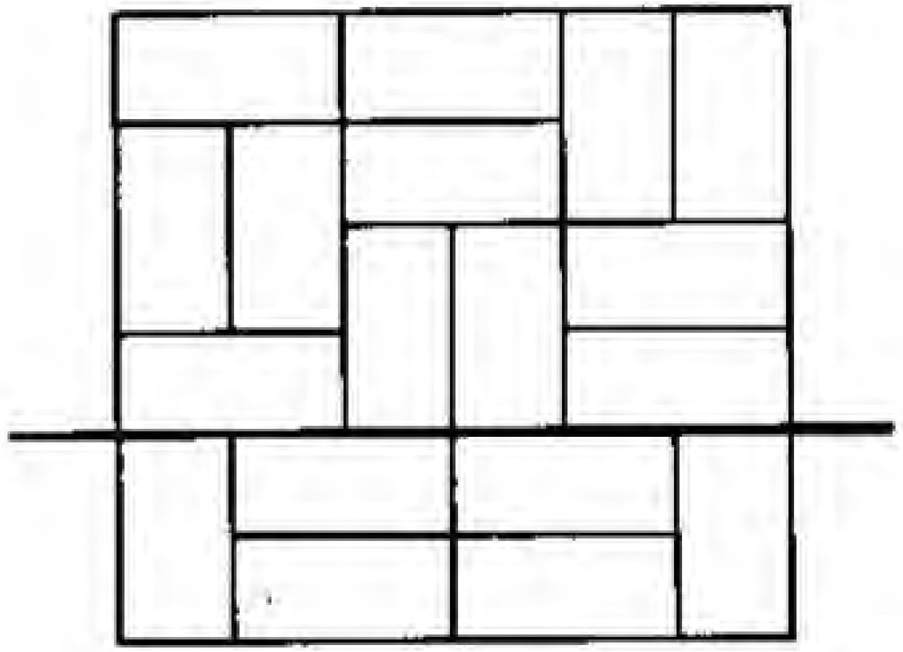




If you succeed, show your solution. If you don't, explain why you could not.

Hints on p. 90 Solution on p.158 Comments on p. 238

76. EIGHTEEN DOMINOES



Procure a 36 square mesh, 6 units by 6 units, as in the previous problem, but without blocking any square, so that it is always possible (in innumerable ways) to lay out 18 dominoes in the mesh. Here, the problem is different. The figure above shows an arrangement of the dominoes, but there is a line drawn straight from one edge of the boundary to the opposite edge without cutting any domino, but passing

between them. However, what is required is an arrangement of the dominoes in which there exists no such straight line (horizontal or vertical).

Hints on p. 90 Solution on p. 159 Comments on p. 238

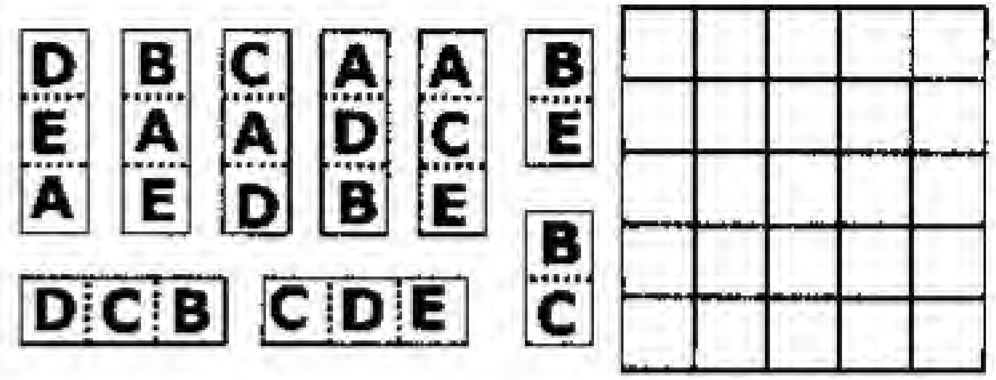
77. FIVE BY FIVE COLOUR SQUARE

This problem requires a little preparation with pieces of thick paper or card. Make the following: A base 5 unit by 5 unit mesh.

Seven trominoes and two dominoes, with which to cover the mesh. (A tromino is a 3 by 1 rectangle and a domino, a 2 by 1 rectangle)

Five different colours, one to paint each unit square will make the thing very attractive. Here, we designate the colours by five different letters A, B, C, D, E. The following figures show the completed trominoes and dominoes.

The problem: Lay out the nine pieces on the mesh so that the colours in each row, column and the two main diagonals are all different.



Hints on p. 91 Solution on p. 160 Comments on p. 238

78. ODD-EVEN ALPHAMATICS

In an actual multiplication of a three-digit number by a two digit number, if every even digit (0, 2, 4, 6, 8) is replaced by E and every odd digit (1, 3, 5, 7, 9) is replaced by O, then the result of the multiplication will be as follows:

		E	E	O
			E	O
		O	O	O
	O	E	E	
O	E	O	O	O

Restore the original multiplication and show that the solution is unique.

Hints on p. 91 Solution on p. 160

79. TWO FRIENDS

Krishnan and Rajesh were good friends. They lived in homes in the same street at a walkable distance of a few meters between them. One day, Krishnan started at 9 o'clock in the morning from home to meet Rajesh, while Rajesh, unaware of his friend's intention, also left home exactly at 9 o'clock in the morning on the same day to meet Krishnan. While walking along the street, Krishnan was deeply engrossed in his thoughts on Wimbledon finals and Rajesh was immersed in his thoughts on his role in the next film. So, when they crossed, they did not see each other, but continued to walk. From the point of crossing Rajesh took 8 minutes to reach Krishnan's house and Krishnan took 18 minutes to reach Rajesh's house. At what time did Rajesh reach Krishnan's house, and Krishnan Rajesh's?

Hints on p. 91 Solution on p. 161 Comments on p. 239

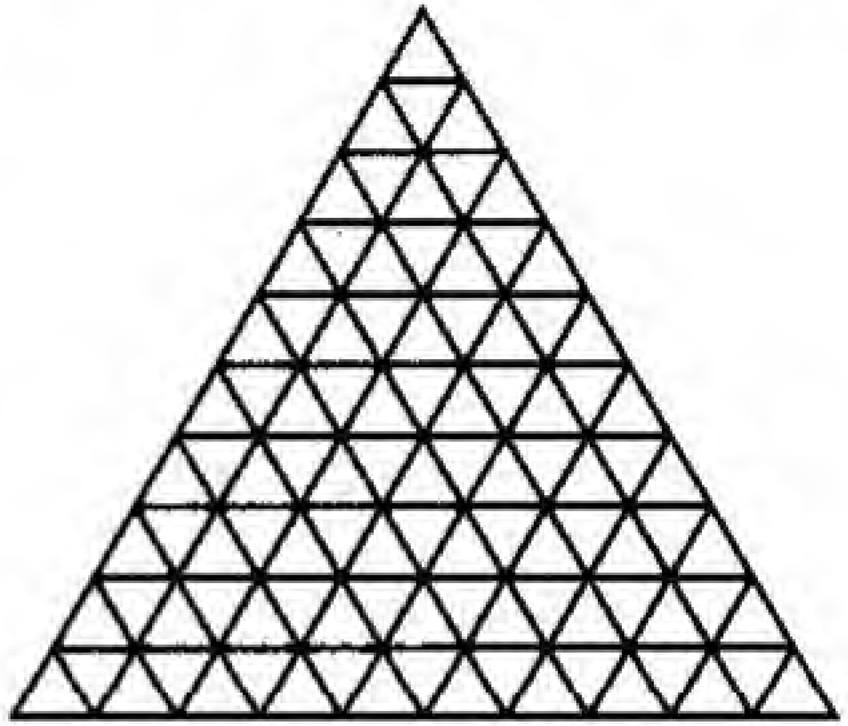
80. PAPER FOLDING

A long strip of paper AB is folded (and pressed to make a crease) so that the end B falls on the strip at a distance *a* from A. After opening out the strip, it is again folded so that the end A falls on the strip at a distance *b* from B. What is the distance between the two creases?

Hints on p. 91 Solution on p. 162 Comments on p. 239

81. TRIANGULAR TIER

How many triangles, big and small, are there in the diagram shown below.



Hints on p. 92 Solution on p. 163 Comments on p. 229

82. ANAND'S AGE

Expressed in months, Anand's age is the square of Abhay's. 51 months later Anand's age will be a multiple of Abhay's. If Anand retired from active service at the age of 720 months, how many months ago did he retire?

Hints on p. 92 Solution on p. 164

83. 1 to 80

1	4	7	10	13	16	19	22	25
28	31	34	37	40	42	45	48	51
54	57	60	63	66	69	72	75	78

2	5	8	11	14	17	20	23	26
29	32	35	38	41	44	47	50	53
56	59	62	65	68	71	74	77	80

2	3	4	11	12	13	20	21	22
29	30	31	38	39	40	45	46	47
54	55	56	63	64	65	72	73	74

5	6	7	14	15	16	23	24	25
32	33	34	42	43	44	51	52	53
60	61	62	69	70	71	78	79	80

5	6	7	8	9	10	11	12	13
32	33	34	35	36	37	38	39	40
54	55	56	57	58	59	60	61	62

14	15	16	17	18	19	20	21	22
45	46	47	48	49	50	51	52	53
72	73	74	75	76	77	78	79	80

14	15	16	17	18	19	20	21	22
23	24	25	26	27	28	29	30	31
32	33	34	35	36	37	38	39	40

54	55	56	57	58	59	60	61	62
63	64	65	66	67	68	69	70	71
72	73	74	75	76	77	78	79	80

Take four strips of blank paper and write 27 numbers shown above on both the sides of each strip. The two sets of 27 numbers shown in the two blocks in the same line are to be written, one set on one side and the other on the other side.

(To preserve them for repeated performances of the trick, it is better to use thick card than ordinary paper.)

Ask a subject, to think of a number from 1 to 80 and let him place on the table only those cards which contain his chosen number so that the side which has the number is below, hidden from view. As the performer, you have a glance at the reverse sides of the cards and announce the chosen number to the astonishment of the subject. How?

Hints on p. 92 Solution on p. 165 Comments on p. 242

84. 2000 A D

On many new year days in the past, as a matter of fun, we showed on our puzzle-board, expressions for the year number using a single-digit a prescribed number of times and arithmetical signs. The last year of last century, coming to a close, was 2000. Let us express this number using a single digit exactly six times and using arithmetical signs. Here are a few examples :

$$2000 = \frac{1 \times 1 \times 1}{.1 \times .1 \times .1} = \left(\frac{4}{.4}\right)^4 \frac{.4}{4} \times \sqrt{4}$$

$$= \sqrt{\left(\frac{6}{.6}\right)^6} + \sqrt{\left(\frac{6}{.6}\right)^6} = \left(\frac{9}{.9}\right)^{\sqrt{9}} \times \frac{9+9}{9}$$

Write down expressions for 2000, using each of the digits 2, 3, 5, 7, 8, one at a time, six times only, and arithmetical signs.

Using arithmetical signs and all the digits 1,2,. . . , 9 in their natural order, we have

$$2000 = 12 + 345 \times 6 + 7 - 89$$

Find an expression for 2000 using the digits 9,8,...,2,1 in the reverse order.

Similarly, we can find expressions for the year 2008, or 2011, except that in first part, a single digit is to be taken eight times.

Hints on p. 92 Solution on p. 166

85. LAST THREE DIGITS

Multiplying two numbers, I find that the last three digits in the product are 147. The last three digits in one of the factors are 729 what are the last three digits of the other factor?

Hints on p. 93 Solution on p. 167 Comments on p. 242

86. BISECTION TIME

After 8.30, there are two moments, when the minute hand of a clock bisects the angle between the hour hand and the vertical line through the centre. What is the time interval between these two moments?

Hints o.· p. 93 Solution on p. 168 Comments on p. 242

87. CALENDAR PUZZLE

Consider the dates in a calendar for any month of any year at the corners of a rectangle of any size. Why is the difference between the products of the diagonally opposite dates always divisible by seven?

Hints on p. 93 Solution on p. 169 Comments on p. 243

88. SQUARE YEAR

1936 was the only year in the twentieth century, which shows a perfect square number. There is only one year in the twenty first century also which has this property. Which year is it? There is one such year in every succeeding century after that until, for the first time, a century will arrive in which there willbe no square number year at all. When will that be?

Hints on p. 93 Solution on p. 170 Comments on p. 243

89. HOW MUCH AREA?

A regular polygon with an odd number of unit sides rotates in its plane about a vertex, making a complete revolution. What is the area swept out by the side remotest from this vertex?

Hints on p. 94 Solution on p. 170 Comments on p. 243

90. EQUAL WEIGHTS

Given nine objects weighing 1 kg, 2 kg.......9 kg and a beam balance, in how many different ways can you place four of the objects in one pan balancing four in the other?

Hints on p. 94 Solution on p. 171

91. IS IT A SQUARE?

Without actually extracting the square root can you find out whether

1827969112905536

is a perfect square number?

Hints on p. 94 Solution on p. 173 Comments on p. 244

92. HANGING ROD

I have a thin uniform heavy rod AB. At what point C on AB should the rod be bent, making angle ACB = 60°, so that when the rod, tied to a string at B, hangs freely, the part AC of the rod remains horizontal ?

Hints on p. 95 Solution on p. 173 Comments on p. 244

93. (10-3) CONFIGURATION

Draw ten lines and mark ten points such that three of the points lie on each line and three of the lines pass through each point.

Hints on p. 95 Solution on p.174 Comments on p. 244

94. DIVISIBILITY TEST

Find a test of divisibility of a number by 7.

Hints on p. 95Solution on p. 175 Comments on p. 245

95. CIRCLES IN CONTACT

Two circles of unit radius touch each other. A third circle of radius two units touches them both internally. What is the radius of a circle touching all the three circles?

Hints on p. 96 Solution on p. 177

96. PERPETUAL CALENDAR

Given the date, month and year (past or future), can you find the day of the week? Is there a formula for this?

Hints on p. 96 Solution on p. 178 Comments on p. 247

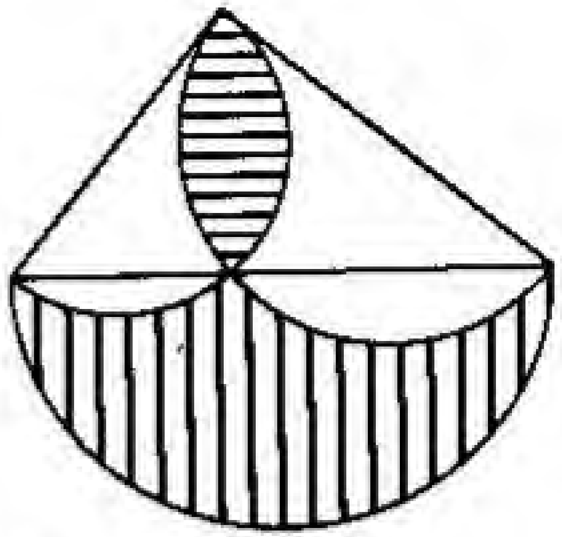
97. CUT NUMBERS

Do you know that the number 6 has the following peculiar property? Add all the numbers before 6, namely 1 + 2 + 3 + 4 + 5 = 15. Add two consecutive numbers after 6. You get 7 + 8 = 15. In the natural sequence of numbers 6 balances (like a fulcrum) all the numbers before it with certain consecutive numbers after it. We call such a number a 'cut number'. The question is: What is the next higher cut number?

Hints on p. 97 Solution on p. 180 Comments on p. 248

98. A CURIOUS RESULT IN AREAS

In the following figure, semi circles have been drawn on the three sides of a right-angled triangle. Show that the area of the triangle is equal to the area of the region with vertical stripes minus the area of the region with horizontal stripes.



Hints on p. 97 Solution on p. 181 Comments on p. 248

99. A HOLE THROUGH A SPHERE

A cylindrical hole is made through a big solid spherical ball. The axis of the cylinder passes through the centre of the sphere and the matter is scooped out. If the height of the cylinder is 12 cms, what is the volume of the remaining part of the ball?

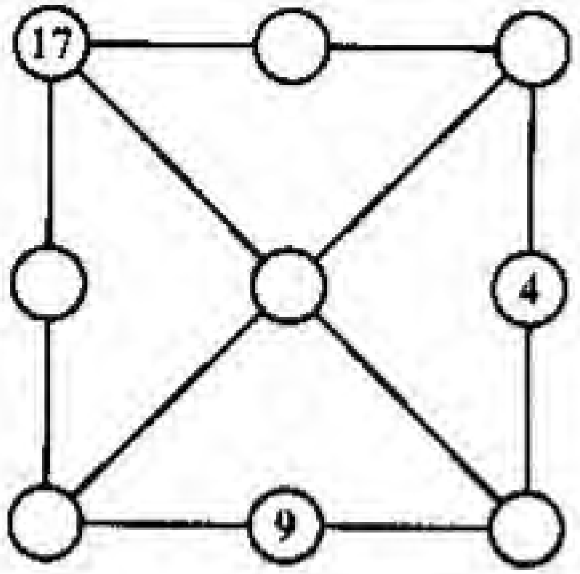
Hints on p. 97 Solution on p. 181 Comments on p. 249

100. AVERAGES

Is the average of a given set of numbers the same as the average of the means of all the pairs of those numbers? (The mean of two numbers is just half of their sum).

Hints on p. 98 Solution on p. 183 Comments on p. 249

101. MEANS AND MISSING NUMBERS



The nine numbers in the nine cells here are such that the number in the middle cell in each of the six lines, is the mean of the numbers at the two ends. Three of the numbers are given. Find the remaining six numbers. (The mean of two numbers is defined as half of their sum, e.g., the mean of 7 and 15 is 11).

Hints on p. 98 Solution on p. 184 Comments on p. 249

102. AREA OF THE FIELD

A rectangular field is divided into four smaller rectangles and the areas of three of them, in square meters, are as shown in the figure. What is the area of the remaining small rectangle?

32	60
24	?

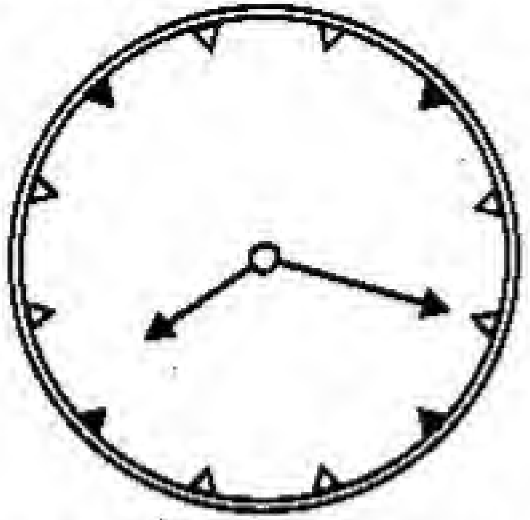
Hints on p. 98 Solution on p. 185 Comments on p. 249

103. WEEK DAY REPEAT

Whatever weekday it is today, it will be the same week day exactly 400 years later. Explain why. For instance, 23rd August, 1882 was a Wednesday. So, 23rd August, 2282 will also be Wednesday.

Hints on p. 98 Solution on p. 185 Comments on p. 250

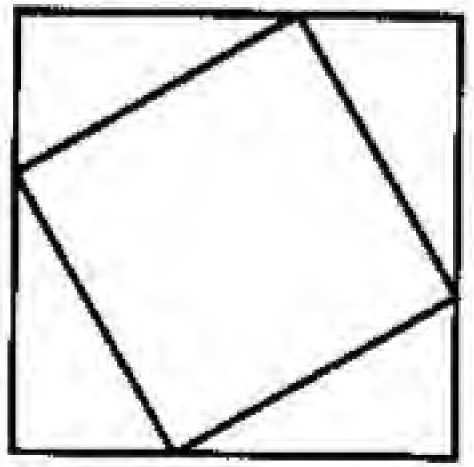
104. TAMPERING WITH TIME



A mischievous boy took out a figureless, symmetric wall-clock, turned it around through 45° in the clockwise sense and hung it up on the wall. Can you, as a visitor, entering the room, look at the image shown above, in a mirror on the opposite wall, and tell the correct time?

Hints on p. 98 Solution on p. 186

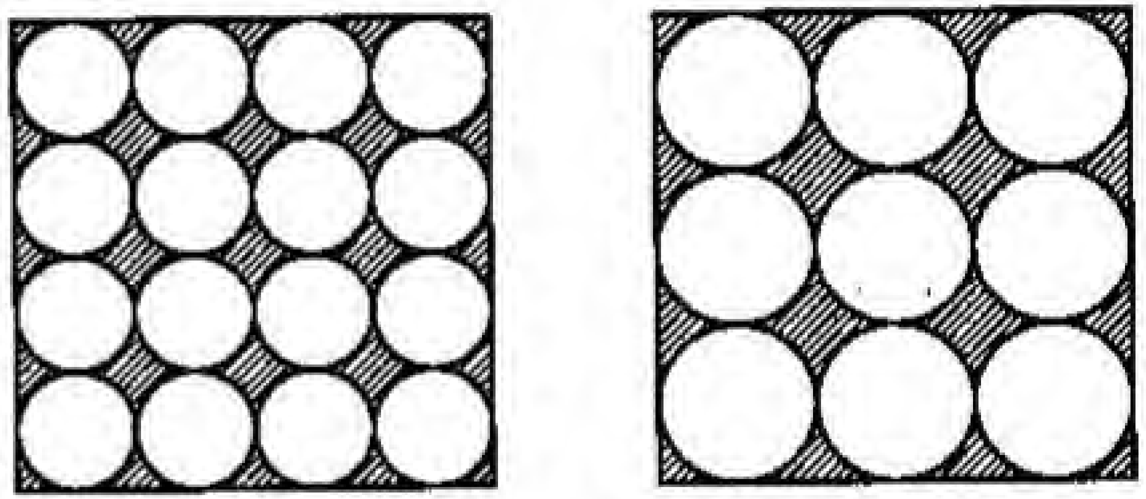
105. SQUARE WITHIN A SQUARE



The figure shows a square within a square. If the area of the bigger square is 1.6 times the smaller one, find the ratio in which a side of the bigger square is divided by a vertex of the smaller one.

Hints on p. 99 Solution on p. 186 Comments on p. 250

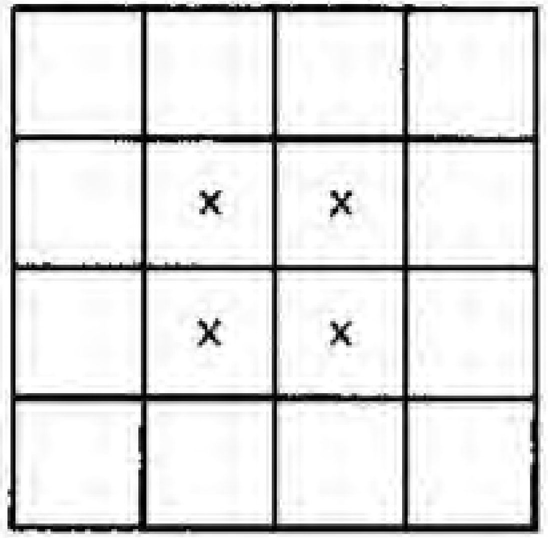
106. COMPARISION OF AREAS



Two equal squares are given as shown in the figure. The question is, which shaded region has larger area OR are they equal ?

Hints on p. 99 Solution on p. 187 Comments on p. 250

107. A CHARACTERISTIC OF 4 x 4 MAGIC SQUARES



In any of the innumerable 4 x 4 magic squares, the sum of the entries in each row, column and diagonal is constant. In any such magic square, prove that the sum of the entries in the four central squares is the same constant sum.

Hints on p. 99 Solution on p. 187 Comments on p. 251

108. SERIES SUMMATION

How many terms of the series $1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 + 9 - 10 + \ldots\ldots$ are to be taken to make the sum equal to 2007 ?

Hints on p. 99 Solution on p. 188 Comments on p. 251

109. OVERTAKING PROBLEM

A champion cyclist riding his bicycle at a uniform speed leaves Ahmedabad at 6.20 a.m. and reaches Vadodara 10 minutes before noon. A race-motorist driving at a uniform speed leaves Ahmedabad at 7.30 a.m. on the same day and reaches Vadodara at 9.20 a.m. At what exact time does the motorist overtake the cyclist ?

Hints on p. 99 Solution on p. 188

110. ADULTERATION

A milkman had 25 litres of milk of which the fat content is 5.4 %. How much water did he add to the milk to reduce the fat content to 5 % ?

Hints on p. 100 Solution on p. 189

111. ROD ON A SEGMENT

A 7 cm. long rod is placed on a horizontal line segment so that the left end of the rod divides the segment in the ratio 1:7 and the right end in the ratio 3:5. Find the length of the segment.

Hints on p. 100 Solution on p. 190 Comments on p. 251

112. SUMS OF SQUARES

65 is a number which can be expressed as a sum of two squares in two different ways, for, $65 = 8^2 + 1^2 = 7^2 + 4^2$. Another example is $125 = 10^2 + 5^2 = 11^2 + 2^2$. Can you find two other such numbers, or at least one more?

Hints on p. 100 Solution on p. 190 Comments on p. 252

113. ROLLING DISC

A circular disc rolls, without sliding, inside a fixed circle of twice the radius and always in touch with it, both lying in a plane. What is the path traced by a point marked on the edge of the disc?

Hints on p. 100 Solution on p. 191 Comments on p.252

114. MULTIPLES OF 11

Using the six digits 1, 2, 3, 4, 5, 6 each exactly once, how many multiples of 11 can we get?

Hints on p. 101 Solution on p. 192 Comments on p. 253

115. PROBLEM OF FOLDING A RECTANGULAR PAPER

A sheet of paper in the form of a rectangle ABCD, of length 20 units and width 15 units is folded and pressed so that the point B falls on and coincides with point D. Find the distance of the new position of C from its old position.

Hints on p. 101 Solution on p. 192 Comments on p. 253

116. NUMBER OF PRIZES

At a prize-giving function, four girls, all friends, received 33 prizes in all. No two of them received the same number of prizes. Considering the girls two by two (that is six pairs), the differences in the number of prizes that they got was 1, 2, 3, 4, 5 and 6, none repeated. Find the number of prizes received by each one of them.

Hints on p.101 Solution on p. 193 Comments on p. 253

117. ARITHMETICAL CONUNDRUM

If TWO times WIT
is five O's, what is IT?

Hints on p. 101 Solution on p. 194

HINTS

1. MILKMAN'S PROBLEM

Get 1 litre of milk. Note that $3 + 3 - 5 = 1$. Then use $1 + 3 = 4$.

2. UNICURSAL FIGURE

Note that four lines emanate from every point of the figure except two from each of which five lines emanate. Start from one of these points and end at the other.

3. NECKLACES

For each number of black beads (less than or equal to 7) consider the several ways of positioning the remaining number of red beads.

4. MAGIC SQUARES

The accompanying standard Magic Square, in which the constant sum is 15 can form the basis of your solution.

8	1	6
3	5	7
4	9	2

5. A RUPEE LOST

Is "Four lemons per rupee and five lemons per rupee" the same as "Nine lemons for two rupees ?" No, not always.

6. ZEROLESS FACTORS

Note that the given number, has nine zeros and so is equal to 10^9 and $10 = 2 \times 5$.

7. FIND THE NUMBERS

Note that the divisor is 2 more than the remainder in each case. So, what is the effect of increasing the required number by 2?

8. AGE PROBLEM

Let x be the present age of my son, and *y* the required number of years - Get the appropriate equation in y and solve.

9. WATCH PROBLEM

The two hands will be perpendicular to each other some time between 3.30 and 3.35. Assume that the exact time required is 30 + x minutes past 3 o'clock. Use the condition of perpendicularity of the two hands.

10. EQUALISE AMOUNTS

Assume that the initial amounts with A, B, C are *a, b,* c. Carry out the transactions as described and equate the final expressions. Your skill lies in finding integral solutions of two equations in three unknowns.

11. LADDER PROBLEM

Assume the required distance as x. Use the fact that the two tangents to a circle from an external point are equal and then use Pythagoras' theorem.

12. WRONG MAN OUT

If you try to copy or carefully trace each figure you may, perhaps, get a clue.

13. MISSING NUMBERS

It is, of course, assumed that some rule is being followed in writing 12 of the numbers in the table. The four numbers in the first row and four numbers in the first column are to begin with, arbitrary. Then each of the other entries is obtained by some simple calculation on only two other numbers. These two numbers are in the first row and first column in the vertical and horizontal line through the chosen entry. You are required to discover the calculation referred to above.

14. ALPHAMATICS

Can you see as obvious the following points in the order given below?

O = 8, D = 2, T + B = 11 + U, R = 0, G + 1 = T. Rest is easy.

15. TWO BOATS

Obviously, each boat took the same time between the two crossings. Hence, the ratio of their speeds is equal to the ratio of the distances they covered.

16. FOUR CHILDREN

Even if you do not know 'Permutations and Combinations', you can solve this simple problem by an actual enumeration. Try.

17. MISSING FIGURE

How many directions on the whole are there of the radii of the several circles? In how many ways can three directions of radii be chosen? Do the actual enumeration.

18. DISSECTION PROBLEM

We have not said that the cut should be a single straight line cut. It is, in fact, a zig-zag with three straight portions.

19. BLACK AND WHITE CAP

Each prisoner could see two black caps on the heads of the other two and on this slender basis alone he cannot conclude that of the remaining two white and a black cap, he himself is having a black cap. As a matter of fact, C has more premises than A and B, and B has more premises than A.

20. WHAT IS "PLUNKY"?

Find out whatever is common to all the four figures in the first column. Then check that no figure in the second column contains the common characteristics in entirety. Thus, you will know what a "Plunky" is.

21. SPECIAL TRIPLET

Represent the six persons by six points. If any two persons are related, join the corresponding points by a black line, otherwise, by a red line. Thus, join all the six points in pairs by 15 lines some of them coloured black and some red. We have now only to show that there are always three points among the six whose three joins are of the same colour.

22. WHO ARE BROTHERS?

Perhaps, you have solved the problem by a lengthy method. But, there is a simple and beautiful piece of reasoning. If one

of the men claims to be a brother of another, what can you say of that other man? Also, if one denies to being a brother of another, what can you say of that other man? Use your answers to these questions as the basis of a short logical chain, to solve the problem completely.

23. DIVISIBLE BY 7

Express the number in terms of A and B, using the place-values of the digits, as powers of 10. An obvious factor will turn out to be divisible by 7.

24. MAKE A SQUARE

Consider the smallest side of the triangle as the unit of length. Find the total area of all the pieces. The square root of this will give the length of a side of the required square. Thus, we know which edges of the pieces should be on the periphery.

25. BIRD AND SNAKE

Draw a figure, assume the point where the bird catches the snake and use Pythagoras' theorem.

26. 12 BALLS PROBLEM

Let us consider here another much simpler problem: "You are given 8 balls all looking absolutely alike and whose weights are perfectly equal, except one whose weight is very slightly heavier than any of the rest. Using a balance only twice, find out the imperfect ball." This is easy and if you wish to solve it yourself, do not read further.

As you might have solved, the solution is as follows. Let the balls be designated as A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8. At first weighing: Put any three balls, say A_1, A_2, A_3 in one pan of the balance and another set of three say, A_4, A_5, A_6, in the other pan of the balance and weigh. Case I: If there is equilibrium, the faulty (heavy) ball is one of A_7 or A_8. In the second weighing, weigh A_7 against A_8 and the pan that goes down contains the faulty ball. Case II: If, in the first weighing, there is no equilibrium, the pan that goes down containing, say A_1, A_2, A_3 will have the faulty ball. In the second weighing, weigh two of these three balls, say A_1 against A_2. If there is equilibrium, A_3, will be faulty. If there is no equilibrium, of A_1 or A_2 whichever goes down is faulty. Easy.

If you still want to solve the original problem of 12 balls yourself, do not read any further.

The 12 balls problem is more difficult. Designate the balls from A_1 to A_{12}. In the first weighing, weigh any four say A_1 to A_4 against any other four say A_5 to A_8. Consider the two cases: equilibrium and non- equilibrium. Proceed further in each case. There may be alternative cases in the second and third weighing also. Note that the faulty ball may be heavier or lighter than a normal ball.

27. PARKING PUZZLE

Use the squares labeled P, R, and O for parking and also the squares B and E when they are available. Any coin left at Q will be a hindrance for the movement of the other coins.

28. MAGIC THIRTEEN

What is the total of all the numbers? How many lines are there and what is the grand total of all the lines? How many times each number is counted in this grand total?

29. EXITS & ENTRANCES

The problem can be solved only if you start from inside some particular rooms, not from any arbitrary place.

30. TROMINO PUZZLE

First try to accommodate 5 trominoes in a 4 x 4 square with any one square deleted.

31. PECULIAR NUMBERS

Property A is almost obvious.

In regard to property B, the fact that the four numbers in the second row are in arithmetic progression is not of much use. But the common difference 14 (= 2 x 7) may give some clue.

Property C is much more subtle.

32. STRANGE ALPHAMATICS

Obviously, the only possible values of N are 0,1,5 and 6. If you visualize the actual steps in the multiplication, it will be easy to dispose off 0 and 1, and with a little more effort, 5 too. T has a limit.

33. DEVOUT PRIEST

Let *x* be the number of flowers the priest started with and *y* the number of flowers he offered in each temple. Obtain an algebraic expression giving the balance of flowers at the end of three offerings. Equating this to zero, we get a single equation in two variables. But, as the number of flowers is integral, there will be a unique least solution.

34. EAST-SOUTH ROUTES

If you are a student of mathematics, you may be able to convert this problem into one on permutations. Otherwise, perhaps, you will have to try out each path. Make sure that the paths are all different.

35. AN UNKNOWN DIGIT

Use the fact that the six-digit number 111111 is divisible by 13.

36. WRONG LABELS

There are only two different ways of labelling the three boxes wrongly.

37. DOUBLE AMOUNT

Assume that Arun and Ashok have x and y rupees respectively. Then express the transactions in the form of equations, solve.

38. PILGRIM SADHU

You have probably solved this problem by drawing the graphs of his journey. Your proof by this method is quite correct. But there is another very simple proof. But to give some hint for this proof, will be like "letting the cat out of the

bag". However, does it really matter on what days the sadhu ascended and descended?

39. FARMER'S WILL

The judge had a cow of his own. After solving the problem or referring to the given solution, please read the comments.

40. A DICE TRICK

You cannot do this unless you are familiar with some conventions regarding dice. A dice is a small cubical object whose six faces have dots on them, one dot on one face, two dots on another, three dots on a third and so on with a maximum of 6 dots on a face. There are many games, which are played with dice. To solve this problem, you must know two important features in a normal marketed variety of dice.

i. The number of dots on each pair of opposite faces total upto 7. That is, 1 is, opposite to 6, 2 to 5 and 3 to 4.

ii. There are two varieties of dice, the left-handed and the right handed, of which only the right-handed is standard and the generally manufactured variety.

41. DIGITS IN A TRIANGLE

In each scheme, the six digits in the top row are arbitrary. But every other digit is in some way related to the two digits just above it.

42. UNICURSAL DESIGN

Note the positions of those points where the straight lines take turns (not where they intersect) and mark them first on your paper.

43. AN AGE PROBLEM

How old is Maya when Dora was born? Subtract 5 and add 3. You get Dora's father's age.

44. REVERSED DIGITS

We have to solve the alphamatics:

$$ABCDE \times 4 = EDCBA$$

As the product is of 5 digits, 'A' must be 1 or 2 as otherwise there will be 6 digits in the product. But as the multiplier is 4 which is even, A must be even. So A = 2. So again, as 4 x E ends in 2, E must be 3 or 8. Now, you may proceed further and show that the solution is unique.

45. PENTAGRAM NUMBERS

Find the constant sum referred to in condition (1). There are fourteen sets of four numbers each satisfying condition (1). From these fourteen sets pick out suitable five satisfying condition (2).

46. CROSS AND SQUARE

Taking an edge of the whole piece as a unit, we see that the area of the cross = 5 sq. units. This is also the area of the required square. Hence the side of the required square = $\sqrt{5}$ units. Now, by Pythagoras' theorem, $\sqrt{5}$ is the length of the hypotenuse of a right-angled triangle whose sides are 1 and 2 units.

47. A COINS PROBLEM

The merchant obviously has 50 ten-paise coins. Let the remaining 50 coins consist of *x*-fifty paise, *y*-one rupee and *z*-five rupee coins.

48. A BRIDGE PROBLEM

This is a geometrical construction problem. You will find it difficult, if you are not a student of geometry. By way of hints, we would first state that if there were no river, the shortest distance between A and B is the straight-line distance. But, actually, the river runs between A and B. Now, imagine the entire half-plane containing B and B-side bank, shifted by a parallel displacement through a distance equal to the width of the river towards the A side bank in a perpendicular direction. This will carry B to a position B'. To distance AB' add the width of the river to get the shortest route.

49. THE MISSING DIGIT

The result of subtraction is always divisible by 9. Then apply the rule of "casting out nines".

50. WATER AND WINE

You may, if you like, assume that half a glass of liquid is *n* times a spoonful, taken as unit measure, determine the measures of water and wine lost from the respective glasses, in terms of *n* and compare the results. But, there is a much easier argument.

51. MAGIC HEXAGON

There are six lines of three numbers each, six lines of four numbers each and three lines of five numbers each. The sum total of each of the fifteen lines must be constant. First, you must find this constant. Then the six other numbers in the empty border cells are easily filled in. A subtle but simple argument is required to fill in the remaining cells.

52. REMAINING REMAINDER

There is a relationship among the three remainders. Try to find this. If you are not a mathematics student, this will not be easy for you. But, if you know some mathematics, we would urge you to find the relationship before having a peep into the solution.

53. A MULTIPLE OF 11

Use the following well-known test of divisibility of a number by 11. Add up the digits in the odd places and then those in the even places separately. If the two sums are equal or differ by a multiple of 11, the number itself will be a multiple of 11. For example, 781935264 is a number with nine different digits and is a multiple of 11, because the sum of the digits in the odd places is 17 and of those in the even places 28 and 28 - 17 = 11. We want the biggest such number.

54. A CHAIN PROBLEM

If your answer is twelve rupees, you are wrong.

55. TWELVE MATCHES

You may think that the pythagorean triplet (3, 4, 5) ($3^2 + 4^2 = 5^2$) has nothing to do with the problem. But, it has something to do with the problem.

56. HOW MANY SQUARES?

There are 70 small squares. How many 2x2 squares are there? How many 3x3 squares? How many 4x4 and so on and lastly how many of the largest 7x7 squares? Add all the results to get the answer.

57. WEDDING INVITEES

This is to be done by solving equations. Use the facts that a bicycle has two wheels, an auto-rickshaw three and a car has four.

58. BUYING COCOANUTS

Do you know the geometrical fact that volumes of similar solids are to one another as the cubes of the corresponding linear dimensions?

59. PROFIT AND LOSS

As selling prices are the same and profit and loss are both 10%,

do you think that on the whole, there is neither profit nor loss? That is wrong. Actually, there is a loss.

60. A LIE

Shantilal must be knowing mathematics. Otherwise, it is not easy to discover that at least one of his children was telling a lie. Do you know that the sum of the series 1 +3 + 5 + 7 +.... to any number of terms is a perfect square?

61. HALF DISTANCE - HALF TIME

The answer is almost obvious. If necessary, you may assume the running speed as u and walking speed as v.

62. PERFECT SQUARE

Convert the largest 5-digit number in base 7 into decimal and find the largest square less than this number. Convert this square back into base 7. The property of a number being a square is independent of the base.

63. RECTANGLE - SQUARE

First find the square, which is equal in area to the given rectangle. Superimpose the square on the rectangle with two of the edges of each coincident. Draw the diagonal of the overlapping region. One of the two congruent triangles

(half of the common region) is one of the pieces. Another piece is also a triangle.

64. RULER CONSTRUCTION

This cannot be done without a good knowledge of geometry. Use Ceva's and Menelaus' theorem and harmonic division of a line-segment.

65. COLOUR CUBES

Note that same cube differently oriented cannot be counted as a different cube. But two cubes, which are mirror reflections of each other, will be considered different. If you get any one of the following answers : 720, 300, 240, 120, 60, 36, and 24 - they are all wrong.

66. MAGIC OCTAHEDRON

An octahedron is an 8-faced, 12-edged and 6-cornered polyhedron. If you do not know this, you must learn to make one out of thick paper or procure one. Then to solve the problem is easy. First, find the constant sum.

67. RECTANGULAR MESH

This is not a unicursal design like the ones in problems 2, 29 and 42. You will have to lift the pencil off the paper only four times. Now, with this information, try to draw the design.

68. KÖNIGSBERG BRIDGES

Can you convert the problem into one of design of network? Actually, the problem has no solution. But our interest lies in proving this fact.

69. A PARADOX

These set of statements constitute a paradox. Taken as a whole system, they are self-contradictory.

70. DIGIT PAIRS

Such a sequence of 10 digits (five pairs) cannot be found. But our interest lies in proving this fact. Suppose the first occurrences of 1,2,3,4,5, are in positions a,b,c,d,e. What are the position values of the second occurences of these digits ? Adding all the position values, we must get 1+2+3+... +10 = 55. What is wrong ?

71. 63 = 64 = 65

Are the triangles, which appear similar in the second and third figures, really similar? If not, what is the inference?

72. CONSTANT SUM

Split each of the 25 numbers in a particular manner. The split numbers are only ten.

73. SQUARE HOLE IN A SQUARE

First form the square with the pieces without a hole. Then, apply displacement to each piece so that the four right angles at the point inside the square go to the four corners of the required bigger square.

74. FOUR 4'S

There is no mathematical method for this. Only trial and error. It is a test of your skill in manipulating numbers.

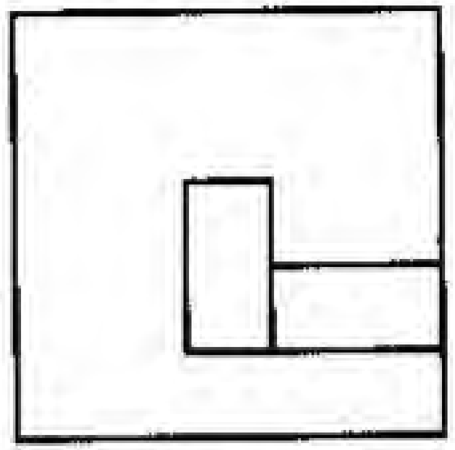
75. SEVENTEEN DOMINOES

A domino always covers two adjacent (horizontal or vertical)squares on the mesh. How are these squares related? If you number the squares of the mesh in a natural number sequence, what will be the parity of the numbers covered?

76. EIGHTEEN DOMINOES

Note that we have to consider only ten lines of this description. How do any of these lines divide the 18 dominoes laid on the mesh?

77. FIVE BY FIVE COLOUR SQUARE



The position of the two dominoes are as shown in the figure.

78. ODD EVEN ALPHAMATICS

It is a matter of simple observation that the O at the left end of the product must be 1. Also the O in OEE must be 9 and so E in the product must be zero. Use parity relationships for addition and multiplication.

79. TWO FRIENDS

Assume their speeds of walking as u and v and then express the distances in terms of u and v.

80. PAPER FOLDING

You may assume the length of the strip as l. But the answer is independent of l, but depends only on a and b.

81. TRIANGULAR TIER

Count the number of erect triangles (Δ) whose sides are of lengths 1,2,.. . , 10. Then count the number of inverted triangles (∇) whose sides are of lengths 1,2.........5. Add all the results.

82. ANAND'S AGE

Let Abhay's age be n months. Then Anand's age will be n^2 months. Express the data algebraically. Knowledge of Diophantine equation is not necessary.

83. 1 TO 80

The fact that the numbers are everywhere in some arithmetical progression will be of no help. On the other hand, choose a few random numbers and note down on what cards they appear in each case, after identifying each face of each card by some means. It may not be easy. But persistence will pay.

84. 2000 AD

As in problem 74, there is no mathematical method here. Just try.

85. LAST THREE DIGITS

Carry out in imagination, the actual multiplication process by writing out the known digits at proper places and blank spaces where the digits are unknown. The unknown digits will reveal themselves one by one.

86. BISECTION TIME

Find the time of each of the two moments separately and take the difference. Use the fact that the minute hand moves through 6° in each minute of time and the hour hand through (1/2)°. Assume in each case, the moment occurs at x minutes past 8 or 8.30, and get an equation in x. Solve for x.

87.CALENDAR PUZZLE

There are 7 days in a week and so the dates in any same column increase by 7.

88. SQUARE YEAR

Use the following facts:

a. The sum of any number of odd numbers in a series starting from 1 is always a perfect square.

b. In the natural number sequence, the difference between two successive squares increases steadily.

c. The gap between two successive squares can certainly widen so much as to sandwich a whole century in a gap.

89. HOW MUCH AREA?

The region swept out is an annular space between two concentric circles. Use Pythagoras' theorem.

90. EQUAL WEIGHTS

Obviously, in each weighing, the unused object is of odd number of kilograms. So, there are five ways of rejecting an object. Use parity laws: Even + even = odd + odd = even, and odd + even = odd, to select four suitable objects to weigh against the remaining four.

91. IS IT A SQUARE?

Add up all the digits of a number N. If the result is a number of more than a digit, add the digits of the sum again. Repeat this process until you get a single digit. This digit is called the digital root of N. Use the idea of the digital root. What is the digital root of the product of two numbers?

92. HANGING ROD

The whole rod should really be regarded as two rods AC and BC rigidly joined at C and so their weights (which are proportional to their lengths) are two vertical forces acting at their middle points. Their resultant must be a vertical force through B, for equilibrium,

93. (10 -3) CONFIGURATION

You may not be able to do this, if you do not know geometry. Use Desargue's Theorem: If the lines joining the corresponding vertices of two triangles are concurrent, then the three points of intersection of the corresponding sides must be collinear.

94. DIVISIBILITY TEST

The hint that we give here is very near the solution. But what we should be really interested in, is the proof that the test will always work. Given a number N_1, remove the last digit of N_1 and subtract twice the digit removed from what remains in N_1, thus getting a number N_2. Prove that if one of N_1 or N_2 is divisible by 7, then the other also is divisible by 7.

95. CIRCLES IN CONTACT

First draw the correct figure. Assume the required radius as x. Apply Pythagoras' theorem and solve for x.

96. PERPETUAL CALENDAR

Take into account the following facts:

1. Assign 1 to Sunday, 2 to Monday..., 6 to Friday and 0 to Saturday. Work with modulus 7, the number of days in a week.
2. Different months have different days. So, assign a suitable permanent index to each month.
3. If the year number is divisible by 4, it is a leap year and February of that year has 29 days.
4. Century years like 1700, 1800, (with two zeros) are not leap year, though divisible by 4, with an exception, namely, if they are divisible by 400, like 1600, 2000, they are to be taken as leap years.

97. CUT NUMBERS

If you do not know a little algebra and number theory, the only way is to solve by trial. But, you can save time by using the formulae

$$1+2+3+\dots+n = \frac{1}{2}\,n(n+1)$$

$$a+(a+1)+\dots \text{ to } n \text{ terms} = \frac{n}{2} \times (2a+n-1)$$

98. A CURIOUS RESULT IN AREAS

Apply Pythagoras' theorem to the figure of a right-angled triangle with circles described on the sides as diameters.

99. A HOLE THROUGH A SPHERE

Surprisingly, we do not say how big the ball is. Use the formulae for the volumes of a sphere, cylinder and segment of a sphere.

Sphere - $\frac{4}{3}\pi r^3$

Cylinder - $\pi r^2 h$

Segment of a sphere - $\frac{1}{3}\pi h^2(3r-h)$

100. AVERAGES

Verify. Take, say, 5 numbers at random. Write out the 10 pairs and their ten means. Add the means and divide by 10. Do you get the same mean as the sum of the 5 original numbers divided by 5?

In the general case, the answer is 'yes'. But we are interested in the proof.

101. MEANS AND MISSING NUMBERS

Assume that one of the empty cells contains *x*. Obtain an equation and solve.

102. AREA OF THE FIELD

For a length or width of the three known rectangles, take a common factor of their areas.

103. WEEK DAY REPEAT

It is easy to calculate the number of intervening days and week days repeat after every seven days.

104. TAMPERING WITH TIME

Can you not turn the figure through 45° in imagination and then visualize its mirror image?

105. SQUARE WITHIN A SQUARE

Note first that the four triangles are congruent triangles. Denote the sides of the triangle by a,b,c (then $a^2+b^2=c^2$). Write down the two areas in terms of a, b, c and form an

equation in a,b. Solve for $(\frac{a}{b})$.

106. COMPARISION OF AREAS

If you take the side of the given square as a unit, what are the areas of the two types of circles ?

107. A CHARACTERISTIC OF 4 x 4 MAGIC SQUARES

Consider the entries in two columns and the two diagonals which involve the four central square entries.

108. SERIES SUMMATION

No mathematical rigour is intended here. On the basis of a close observation of a pattern in the sequence of sums, you can find the answer easily.

109. OVERTAKING PROBLEM

The problem is easily solved by assuming the speeds of both, obtaining equations and solving them. But we

challenge you to solve the problem without using algebra, and this is what we are giving in our solution.

110. ADULTERATION
Here, too, no algebra is required. It is easy enough by simple arithmetic.

111. ROD ON A SEGMENT
By assuming the uncovered parts of the line segment, as x and y, you will get two equations to be solved. But before you look at the solution, we challenge you to solve this problem without algebra.

112. SUMS OF SQUARES
This can be solved by transposition and using the identity
$a^2 - b^2 = (a+b)(a-b)$

113. ROLLING DISC
We recommend performing the experiment as a rewarding experience and is worth the trouble of procuring a small disc and a ring of twice the radius.

114. MULTIPLES OF 11
Use the test of divisibility by 11 : The sum of the digits in the odd places and the sum of those in the even places, must be equal or differ by a multiple of 11.

115. PROBLEM OF FOLDING A RECTANGULAR PAPER

Calculate the length of the perpendicular from C to the line of crease. Twice this length is the answer.

116. NUMBER OF PRIZES
If you have four numbers with the differences as required, adding any fixed number to each of them, will not change the differences.

117. ARITHMETICAL CONUNDRUM
This is actually an alphamatic. Replace each letter by a digit in the following

TWO x WIT = OOOOO.

Note that O is a factor on the right hand side and it occurs in TWO. Also, T and W are repeated.

SOLUTIONS

1. MILKMAN'S PROBLEM

The transfers of milk are to be made as indicated in the table.

8	5	3
8	–	–
5	–	3
5	3	–
2	3	3
2	5	1
7	–	1
7	1	–
4	1	3
4	4	–

Draw a parallelogram having two sides of length 3 and 5 units and one angle of measure 60^0. Take points on it as shown in the

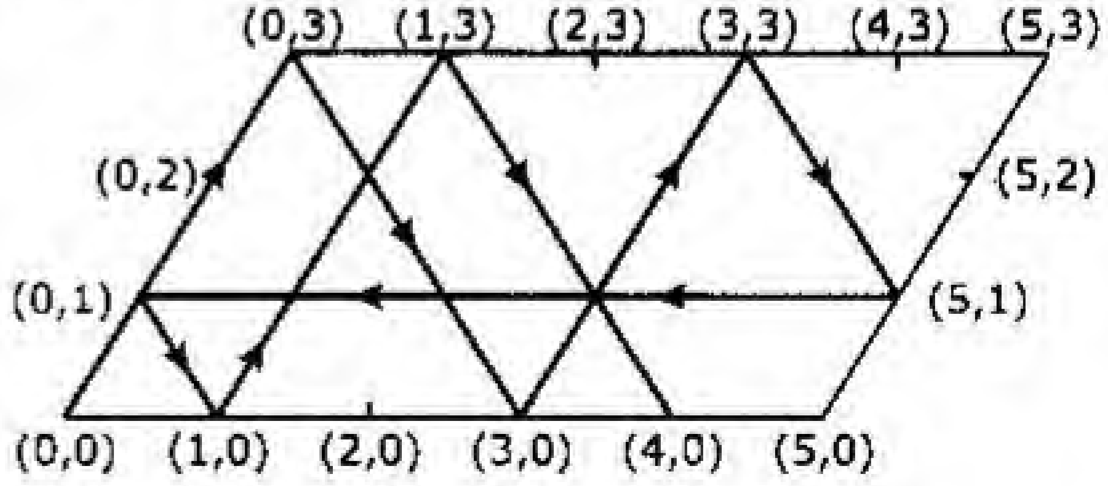
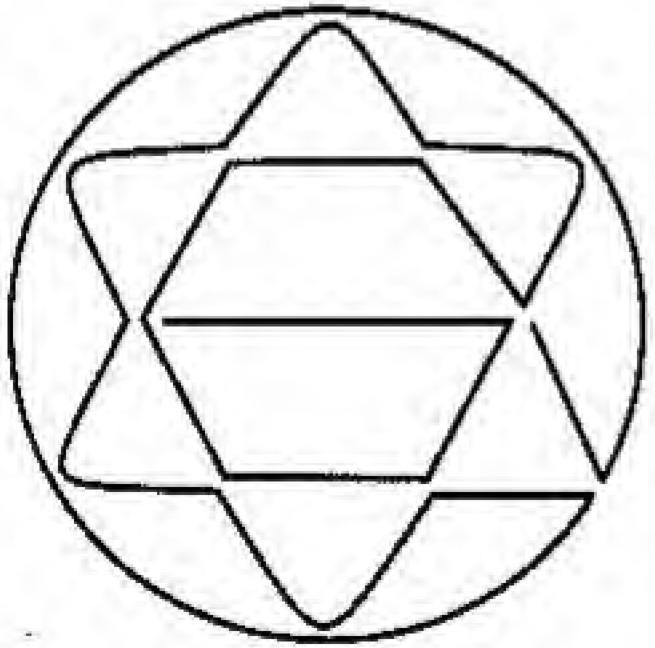


figure and denote them by pairs of points, as in oblique coordinate system. Imagine a ray of light, starting at (0,0) and moving towards (1,0), being reflected by the sides of the parallelogram, one after the other. Stop when you come across 4 for the first time. Take both the coordinates of the points where reflections take place and write them down in columns. You will be surprised to see that the columns which we have written are precisely the second and third columns in the first table. The first column gets filled up automatically. Really speaking, we do not need to think at all here.

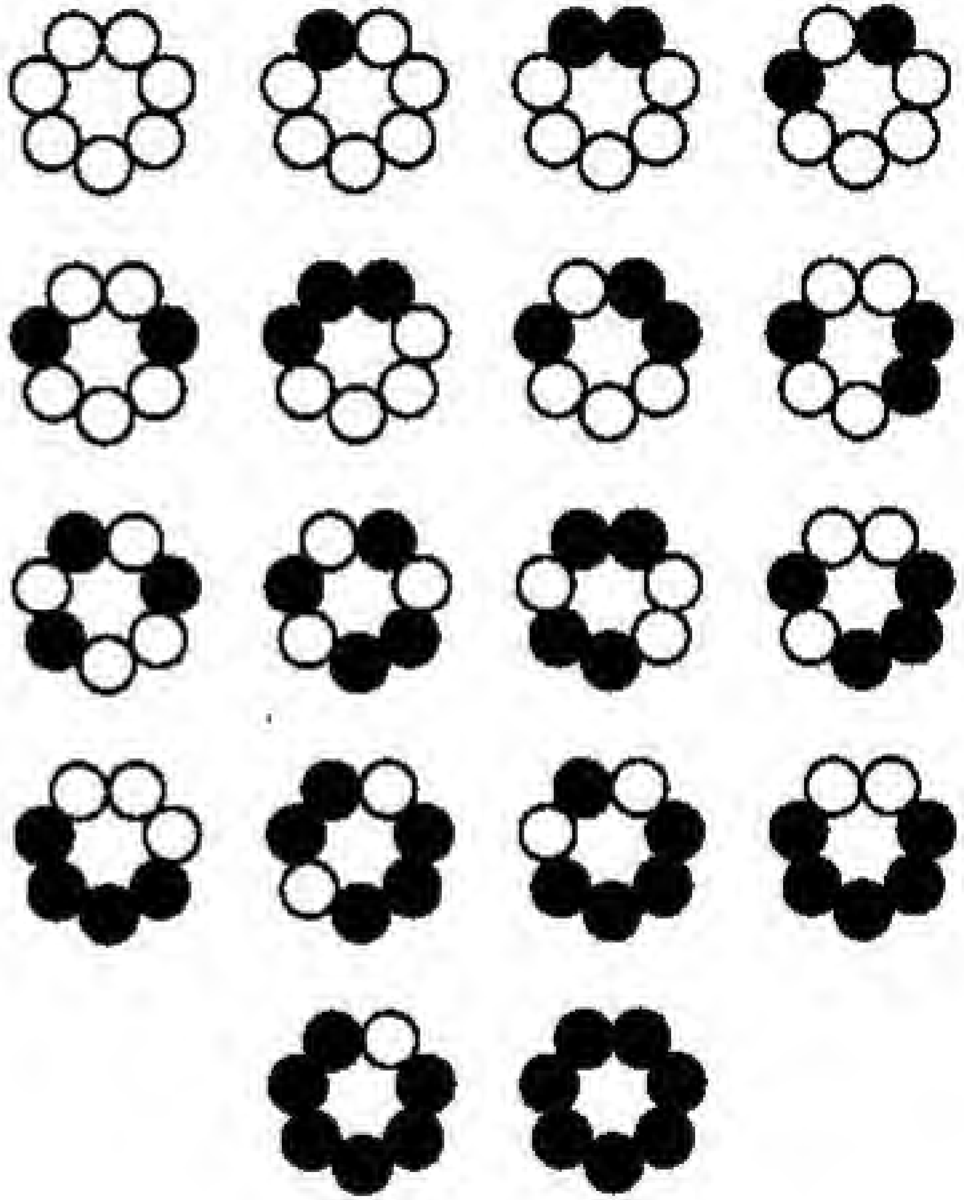
2. UNICURSAL FIGURE

The solution is shown in the figure.



3. NECKLACES

There are 18 different necklaces, as shown in the figure below, 63 beads of each colour will be required.



4. MAGIC SQUARES

The constant sum has to be 105 instead of 15, that is 90 more. As each sum is the sum of three numbers in a line, divide this 90 by 3 getting 30. So add 30 to each entry in the standard magic square. Thus you get one solution. The other solution is obtained by replacing the entries in the standard square by corresponding numbers in an arithmetical progression. As there are 9 entries we divide 105 by the constant sum 15 getting 7, as the common difference of the A.P. So, the other solution is as shown.

38	31	36
33	35	37
34	39	32

56	7	42
21	35	49
28	63	14

5. A RUPEE LOST

"4 lemons per rupee and 5 lemons per rupee" is the same as "9 lemons for 2 rupees" only as long as the amount collected on behalf of both hawkers is the same. So, after selling lemons at the combined rate for 36 + 36 = 72 rupees, X must sell the remaining lemons at his own rate of 4 per rupee, and not at 9 for 2 rupees. This accounts for the difference of one rupee.

6. ZEROLESS FACTORS

$$1000000000$$

$$10^9 = (2 \times 5)^9 = 2^9 \times 5^9$$

$$= 512 \times 1953125$$

7. FIND THE NUMBER

Adding 2 to the required number, it will become exactly divisible by 3, 5, 7, 9. The L.C.M. of these numbers is 315. So, the required number is $315n - 2$, where n is the least number, which will make $315n - 2$ divisible by 11. As $308n$ is divisible by 11, we have $7n - 2$ divisible by 11. Obviously, $n = 5$. Thus, the required number is $315 \times 5 - 2 = 1573$.

8. AGE PROBLEM

Let x be my son's present age. Four years hence his age will be $x + 4$. Twice this is $2x + 8$. This was my age 4 years ago. So my present age is $2x + 12$. y years hence my and my son's age will be respectively $2x + 12 + y$ and $x + y$. We must have $2x + 12 + y = 2x + 2y$, which gives $y = 12$.

9. WATCH PROBLEM

Each division round the rim of the watch is traversed by the minute hand in one minute. As the hands are at right angles, the hour hand must have moved through x divisions after 3 o'clock, in $30 + x$ minutes. But the hour hand moves through 5 divisions in one hour or 60 minutes, that is, through 1 division in 12 minutes. Therefore,

$12x = 30 + x$, which gives $x = \frac{30}{11} = 2\frac{8}{11}$. Thus, the exact time required is $32\frac{8}{11}$ minutes past 3.

10. EQUALISE AMOUNTS

Let a, b, c, be the amounts with A, B, C, respectively initially. Then, the transactions are as shown below:

A	B	C	
a	b	c	Initially
$a-\frac{1}{2}b-1$	$\frac{3}{2}b+1$	c	After 1st transaction
$a-\frac{1}{2}b-1$	$\frac{3}{2}b-\frac{1}{2}c$	$\frac{3}{2}c+1$	After 2nd transaction
$\frac{3}{2}(a-\frac{1}{2}b-1)+1$	$\frac{3}{2}b-\frac{1}{2}c$	$-\frac{1}{2}a+\frac{1}{4}b+\frac{3}{2}c+\frac{1}{2}$	After 3rd transaction

Multiplying by 4 throughout and equating, we have

$$6a - 3b - 2 = 6b - 2c = -2a + b + 6c + 2$$

which may be written as

$$6a - 9b + 2c - 2 = 0$$

$$2a + 5b - 8c - 2 = 0$$

Eliminating c, $26a - 31b - 10 = 0$

or, $26(a + 2) = 31(b + 2)$

that is, $\frac{a+2}{31} = \frac{b+2}{26} = t$, say

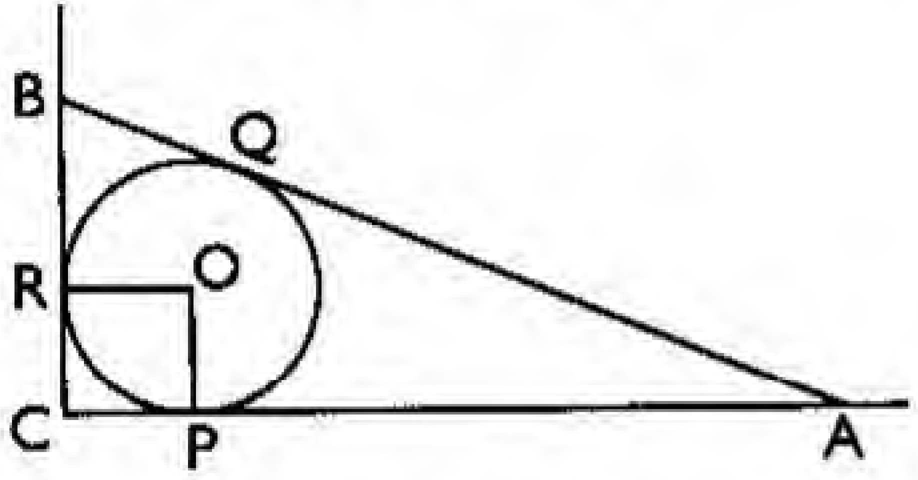
So, $a = 31t - 2$ and $b = 26t - 2$

and substituting in one of the above equations and simplifying, $c = 24t - 2$.

Thus, the least amounts with A, B, C are obtained by taking $t = 1$. So, we get $a = 29$, $b = 24$, $c = 22$.

11. LADDER PROBLEM

The figure shows a section by a vertical plane perpendicular to the wall. Clearly, CPOR is a square whose side is 1 meter. Take $CA = x$. Then $AP = x - 1$. So, $AQ = x - 1$. Hence, $BR = BQ = 6.5 - (x - 1) = 7.5 - x$. Therefore, $CB = 1 + 7.5 - x = 8.5 - x$. By Pythagoras' Theorem, $CA^2 + CB^2 = AB^2$



Hence, $x^2 + (8.5 - x)^2 = 6.5^2$

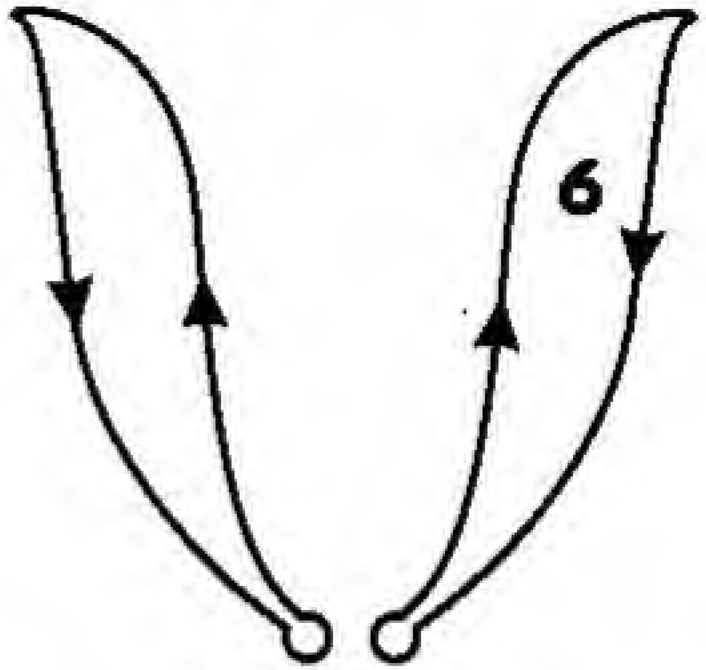
Simplifying, $2x^2 - 17x + 30 = 0$

Solving $(x - 6)(2x - 5) = 0$.

From this, the greater value of x is 6.

Answer: 6 meters.

12. WRONG MAN OUT



Trace, round each figure starting from what looks like a beak and going along the straight edge. In each figure, you will be moving in an anticlockwise direction, except in figure 6, in which the direction is clockwise.

13. MISSING NUMBERS

The calculation referred to in the Hints is as follows. Choose any square. Let the numbers in the first row and first column in the vertical and horizontal lines from the chosen square be a and b. Then the entry in the square chosen is $ab - a - b$. It will be found that all the 12 entries shown obey this rule. Applying this rule to the vacant squares, the entries therein will be 9,41,23,19.

14. ALPHAMATICS

As O $\neq$ R, O > 7 and so O = 8. Considering E in the two places where it occurs, we have D = 2. As G $\neq$ T, R = 0 not 1, since 3 cannot be 'carried over' from the second column, only 2. Hence, G + 1 = T and T + B = U + 11. From the remaining digits 1, 3, 5, 6, 7 a consecutive pair (T, G) must be (6, 5) or (7, 6). If T = 6, then B = U + 5 which appears impossible. So, T = 7, G = 6 and this gives U = 1, B = 5 leaving 3 as the only digit for E. Hence the solution is:

$$\begin{array}{r} 6882 \\ 78 \\ \underline{53} \\ \mathbf{7013} \end{array}$$

15. TWO BOATS

Let us take the distance between the banks along the line of sailing as unit. Then the distances covered by the two boats between the two crossings are $\frac{1}{3}+\frac{1}{2}$ and $\frac{2}{3}+\frac{1}{2}$, that is $\frac{5}{6}$ and $\frac{7}{6}$. So, the ratio of the two speeds is 5 : 7.

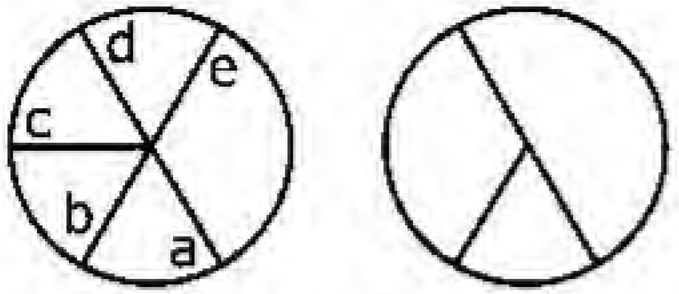
16. FOUR CHILDREN

Let A, B, C, D be the four children. There are only three ways of disposing off (not distributing) eight rupees. They are (a) two children get 3 rupees each and the remaining two 1 each, (b) one child gets 3, two others 2 each and the remaining child 1 rupee, (c) each child gets 2 rupees. It can be seen from the enumeration below that the disposal (a) corresponds to 6 ways of distribution (b) corresponds to 12 ways of distribution and (c) corresponds to only one way. Thus, the total number of ways of distribution is 6 + 12 + 1 = 19.

A	B	C	D
	(a)		
3	3	1	1
3	1	3	1
3	1	1	3
1	3	3	1
1	3	1	3
1	1	3	3
	(b)		
3	1	2	2
1	3	2	2

A	B	C	D
	(b)		
3	2	1	2
1	2	3	2
3	2	2	1
1	2	2	3
2	3	1	2
2	1	3	2
2	3	2	1
2	1	2	3
2	2	3	1
2	2	1	3
	(c)		
2	2	2	2

17. MISSING FIGURE



Notice that each figure consists of a circle with radii whose directions are chosen from the fixed directions as shown here.

Designate these directions as a, b, c, d, e. There are just ten ways of choosing three things from five. These are as follows.

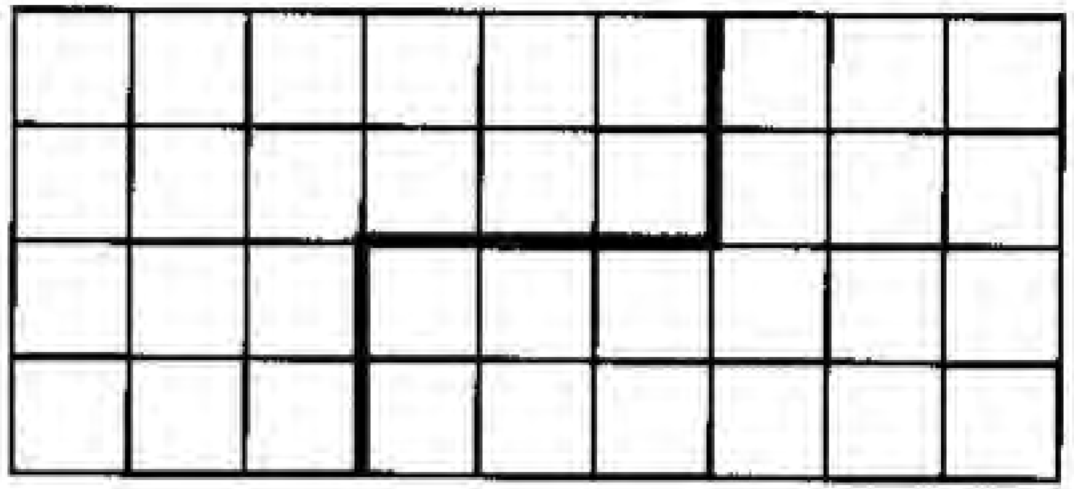
(1)	a	b	c	(6)	a	d	e
(2)	a	b	d	(7)	b	c	d
(3)	a	b	e	(8)	b	c	e
(4)	a	c	d	(9)	b	d	e
(5)	a	c	e	(10)	c	d	e

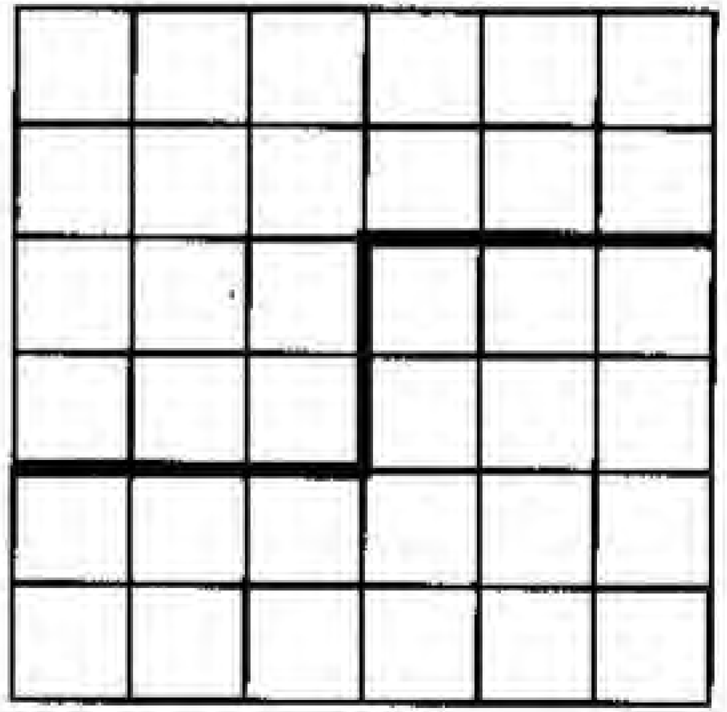
The given figures respectively correspond to the directions:

5	9	10
4	1	8
3	7	6

So, the missing item is (2) a b d. The corresponding figure is shown above.

18. DISSECTION PROBLEM





The solution is shown in the figure above, which is self-explanatory.

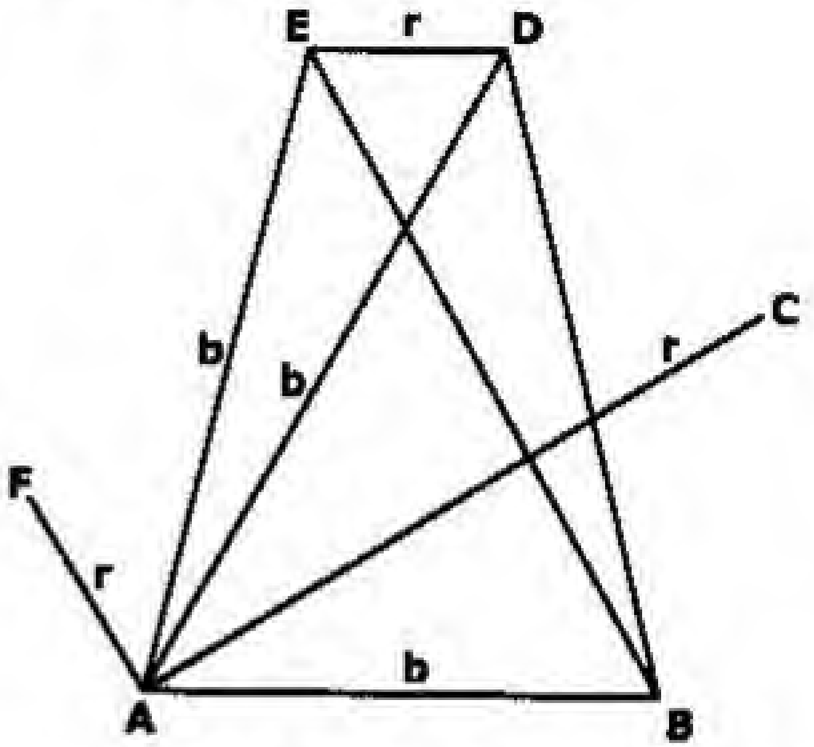
19. BLACK AND WHITE CAPS

If any one prisoner saw two white caps on the others, he would have, of course, concluded that he had a black cap. That was actually not the case. But, C argued by 'reductio ad absurdum' thus: Suppose I have a white cap. I can come to a contradiction. For, B seeing my white cap could have concluded that he is not wearing a white cap, because A said "I don't know". But, since B also said, "I don't know", my cap cannot be white.

20. WHAT IS A "PLUNKY"?

The minimum characteristics of a "Plunky" are shown in the figure. Any additional characteristic is just an embellishment. With this meaning of a plunky, each figure in the first column is a plunky. None in the second column is a plunky. Obviously, of the figures in the third column, the first and third figures only are plunkies.

21. SPECIAL TRIPLETS



Referring to the Hints, let A, B, C, D, E and F be the six points. Consider any one point, say A and the five lines from A, namely AB, AC, AD, AE, AF. However these may be coloured, it is obvious that at least three of them are of one colour. For definiteness, let us say that AB, AD, AE are black and AC, AF may be red or black. Now if the sides of the triangle BDE are all red, then we have proved the proposition. If they are not all red, at least one of them will be black, say BE. Then, ABE will be a triangle all of whose sides will be black. So, in any case, whatever may be the scheme of colouring the lines, red or black, there will always be a unicolour triangle and this completes the proof.

22. WHO ARE BROTHERS?

Let us call a man *t* or *f* according as he speaks the truth or falsehood, respectively. Now whether C is an *f* or a *t* as he claims that F is his brother, F can only be *t*. This is the subtle reasoning mentioned in the Hints. Check it up. Similarly, whether D is an *f* or a *t*, as he denies being a brother of A, A must be an *f* only. Now F speaks the truth when he says that A and E are brothers. So, E is an *f*. A says that B and F are brothers and we know that A lies. So, as F is a *t*, B must be an *f*. Again, B says that D and E are not brothers and B lies. But E is an *f*. Hence, D is an *f*. Lastly, E says that B and C are brothers and we have shown that E lies. Thus, as B is an *f*, C must be a *t*. So we have fixed every man and the answer is: C and F are brothers, as both are *t's*. The remaining four A, B, D, E belong to the other category of brothers all being *f*.

23. DIVISIBLE BY 7

Our number $= A10^5 + B\,10^4 + A\,10^3 + B\,10^2 + A\,10 + B$

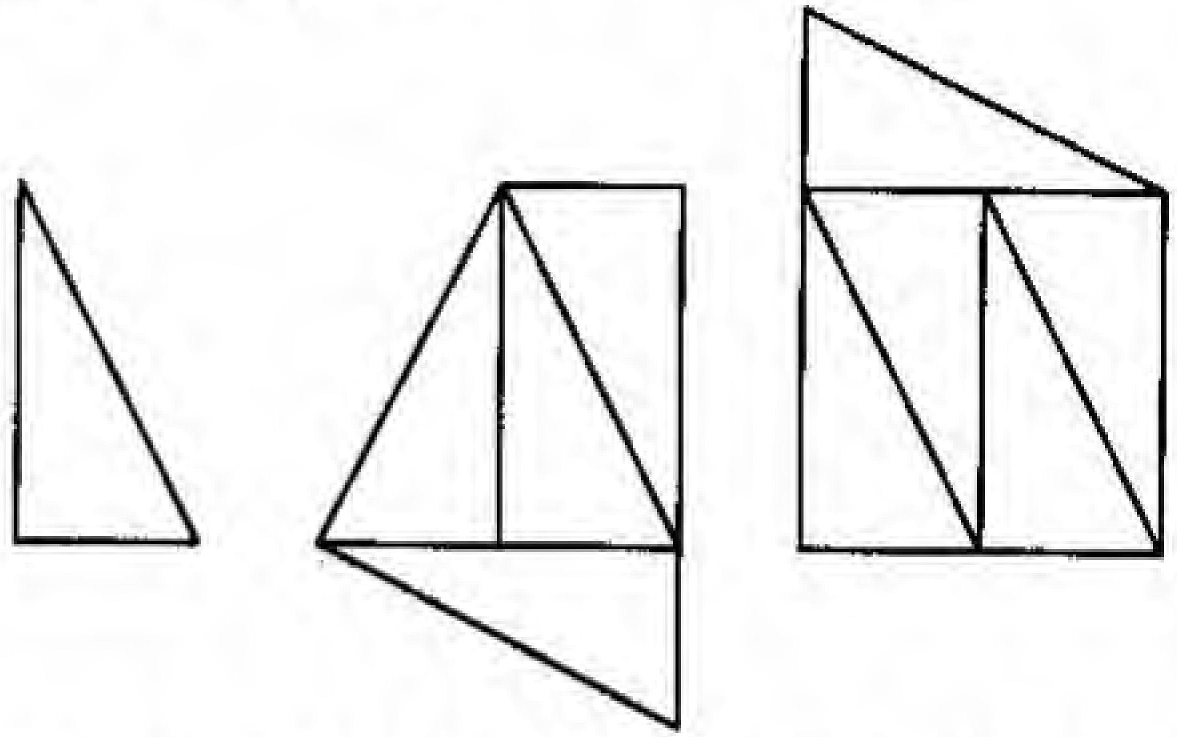
$$= A(10^5 + 10^3 + 10) + B(10^4 + 10^2 + 1)$$

$$= 101010A + 10101B$$

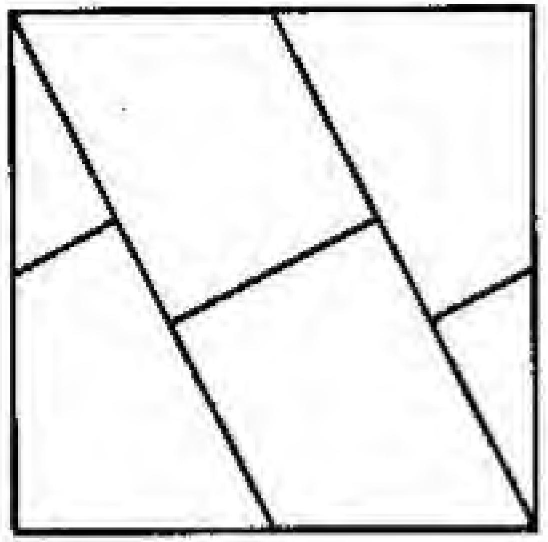
$$= 10101(10A + B)$$

$$= 7 \times 1443(10A + B)$$

24. MAKE A SQUARE



Take the shortest side of the triangular piece as 1. Then the other side of this piece is seen to be 2. So by Pythagoras' Theorem, the hypotenuse is $\sqrt{5}$ units. The area of a triangular piece is 1 sq. unit. The trapezium piece can be seen to contain just 5 triangles and the quadrilateral piece 4 triangles. Thus, the total area of the required square is 20 sq. units. Hence a side of the square is $\sqrt{20}$ or $2\sqrt{5}$. Therefore, the perimeter is $8\sqrt{5}$. It can be seen that there are just 8 edges among the pieces of $\sqrt{5}$ length each. These must be on the periphery and the rest of the edges will go inside the square. The solution is shown on the next page.



25. BIRD AND SNAKE

Suppose the bird catches the snake at a distance of x meters from the foot of the tower. So the snake covers a distance of $(100 - x)$ meters. As the two speeds are equal the bird also covers $(100 - x)$ meters. Hence, by Pythagoras' Theorem,

$$(100 - x)^2 = 20^2 + x^2$$

So, $10000 - 200x + x^2 = 400 + x^2$

Hence, $200x = 9600$

Giving, $x = 48$

Answer: 48 meters.

26. 12 BALLS PROBLEM

Refer to the Hints. In the first weighing, A_1, A_2, A_3, A_4 are in one pan of the balance and A_5, A_6, A_7, A_8 in the other. Case I: Equilibrium. A_1 to A_8 are all normal balls. The faulty ball is among A_9, A_{10}, A_{11}, A_{12}. In the second weighing, weigh any three of these say A_9, A_{10}, A_{11}, against three normal balls say A_1, A_2, A_3. Case I(i). Equilibrium. Conclusion: A_{12} must be faulty. A third weighing of A_{12} against a normal ball will reveal if A_{12} is heavier or lighter than normal. Case I(ii) Non-equilibrium, (a) suppose the balls A_9, A_{10}, A_{11} go down. As A_1, A_2, A_3 are normal, the faulty ball is heavier than normal and is one of A_9, A_{10}, A_{11}. It is easy to find which of these is faulty in the third weighing as in the hints, (b) If the balls, A_9, A_{10}, A_{11} go up, we may track down the faulty (lighter) ball in a similar way.

Case II. Non-equilibrium. Now, A_9, A_{10}, A_{11}, A_{12} are normal. Faulty ball is one of A_1 to A_8. For definiteness, suppose A_1, A_2, A_3, A_4 go down. In the second weighing, weigh A_1, A_2, A_5 against A_3, A_4, A_6. Case II (i) Equilibrium. The faulty ball is one of A_7 or A_8 and so it is lighter. In the third weighing weigh A_7 against A_8 and the one which goes up is faulty. Case II (ii) Non-equilibrium. Suppose A_1, A_2, A_5 go down. A_5 which went up in the first weighing goes down in the second and so cannot be faulty. The faulty is one of A_1, A_2, A_6. In the third weighing weigh A_1 against A_2. Case II (ii) (a) Equilibrium.

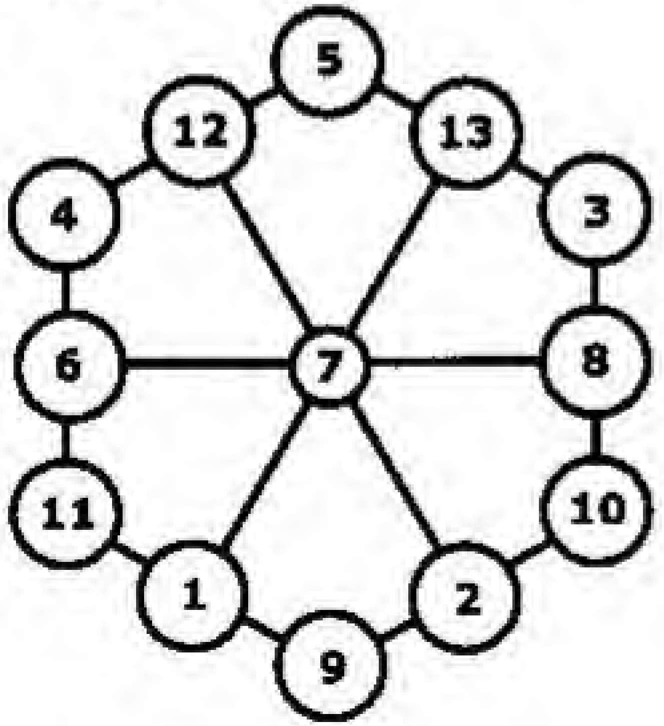
A_6 is faulty and it is lighter. Case II (ii) b. Non-Equilibrium. The ball, which goes down, is faulty and it is heavier than normal. In the first weighing, the case where A_1, A_2, A_3, A_4 go up can be similarly discussed.

27. PARKING PUZZLE

In the solution given below each pair of letters indicates a move of a coin from the first letter to the second.

BO ----- EB ----- DP ----- OD ----- PE
BR ----- CO ----- RC ----- EB ----- FP
OF ----- PE ----- BR ----- AO ----- RA
EB ----- OE

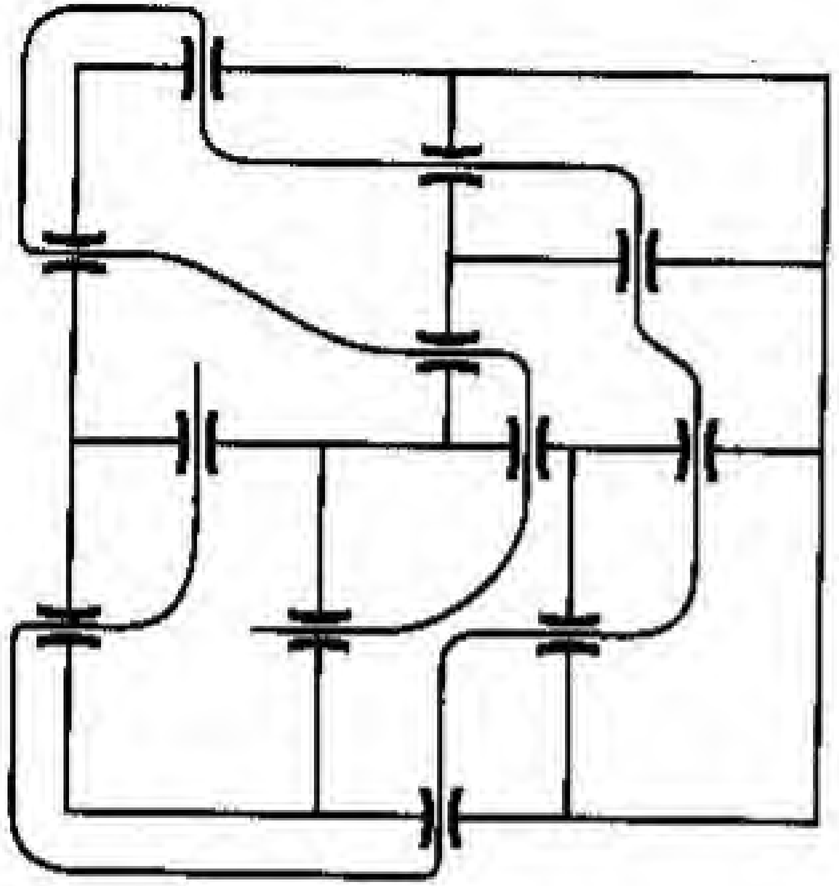
28. MAGIC THIRTEEN



The sum of the numbers from 1 to 13 is 91. Let x be the-constant sum of three numbers in any one line. Then, as there are 9 lines, $9x$ will be the grand total of all the lines. But in this each number has been counted twice except the one at the centre, say a, which has been counted thrice. Hence $9x = 2 \times 91 + a = 182 + a$. So, $182 + a$ must be divisible by 9.

Therefore, a can only be 7. After placing 7 at the centre, it is not difficult to place numbers on the periphery. First, note that $x = 21$. Start by placing 13 in any circle on the periphery. What pair of numbers together with 13 will make 21? The only possibilities are 6,2 or 5, 3. Take either pair and proceed further. One solution is shown here. Can you find others?

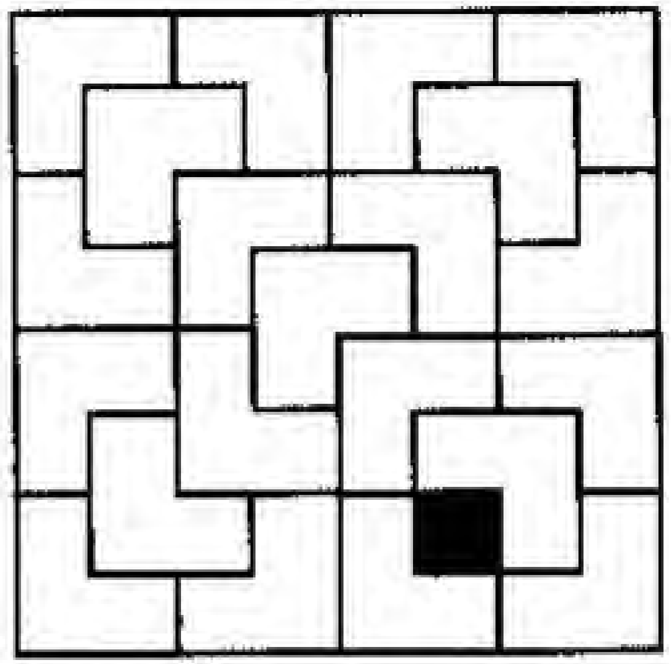
29. EXITS & ENTRANCES



The figure shows one of the paths you can take.

30. TROMINO PUZZLE

The figure on the next page shows the solution.



31. PECULIAR NUMBERS

Property A : Even numbers.

Property B : Leaves remainder 2 when divided by 7.

Property C : 1 more than a perfect square.

32. STRANGE ALPHAMATICS

If N=0, then E is also 0 and so we reject this case. If N=1, E must be 0 and the second digit in the product DOZEN will also be 0 and clashes with E. So N=1 is rejected. If N=5, the fourth digit in the product, namely E, is the last digit of 10E+2,

and so E=2. Now T cannot be 4 or more. For otherwise, the product will contain 6 digits instead of 5. As E=2, T cannot even be 3. As T $\neq$ 2, T=1 is the only possibility. But it is easily verified that in 125 x 125=DOZEN, O is 5 and clashes with N. $\therefore$N=5 is rejected. So ultimately, N=6. In this case, the last digit of 12E+3 must be E, which means that the last digit of 11E+3 must be 0. $\therefore$ E=7. Now T cannot be 3. It is easily verified that in 276 x 276 = DOZEN, O is 6 and clashes with N=6. $\therefore$ we are finally left with the unique solution

$$176 \times 176 = 30976.$$

33. DEVOUT PRIEST

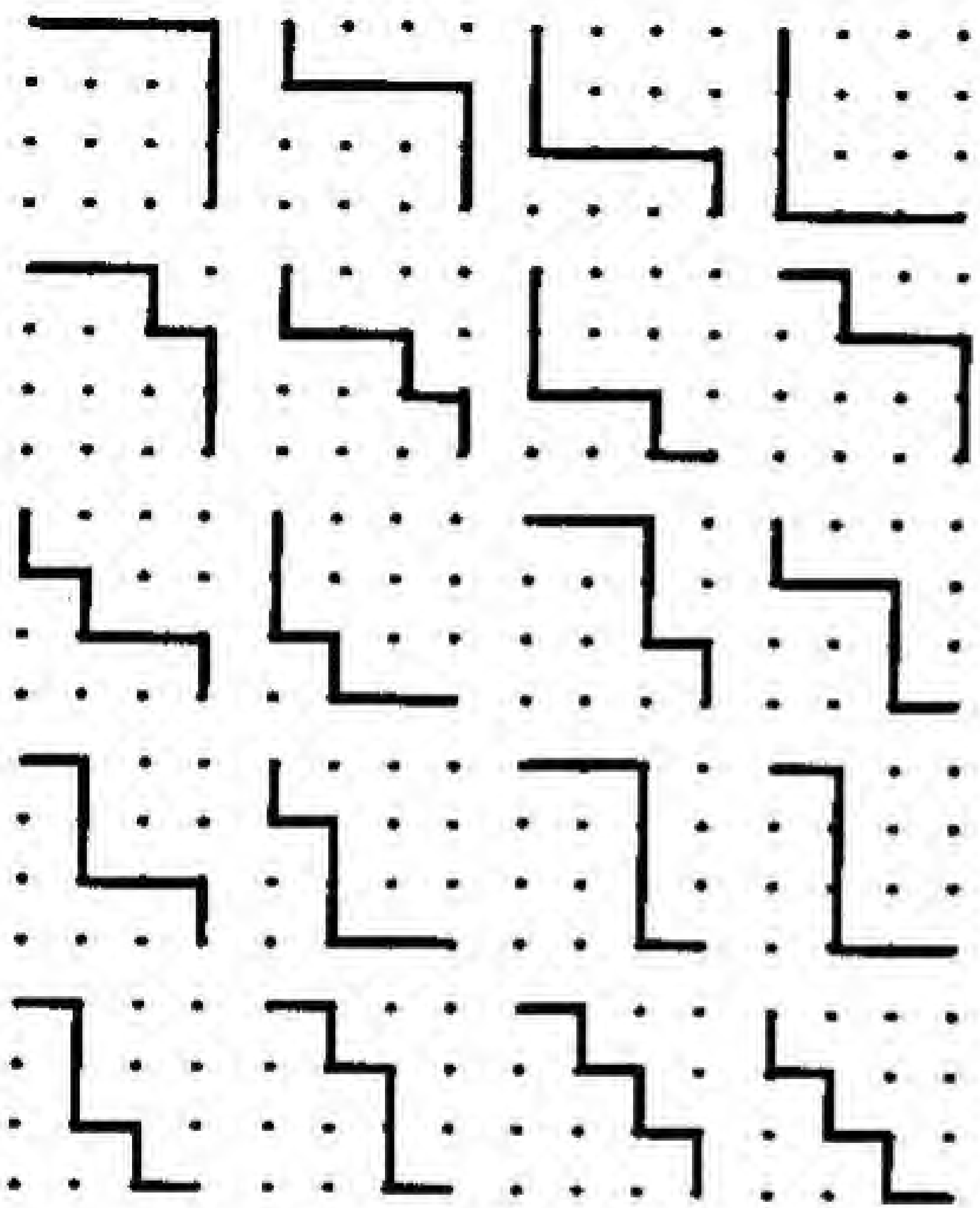
Referring to the Hints, after dipping the basket in the first pond, the number of flowers will be $\frac{3x}{2}$. Of this, the priest offers y in the first temple leaving $\frac{3x}{2} - y$ flowers in the basket.

This, after dipping in the second pond, becomes $\frac{3}{2}(\frac{3}{2}x-y)$ flowers which is $\frac{9}{4}x - \frac{3}{2}y$. Subtracting y from this for the second offering, we get $\frac{9}{4}x-\frac{5}{2}y$ flowers before the last dipping. Therefore, the flowers in the basket after the third offering will be $\frac{3}{2}(\frac{9}{4}x-\frac{5}{2}y)-y=\frac{27}{8}x-\frac{19}{4}y$.

Equating this to zero, we get $27x - 38y = 0$. It is obvious that the least integral values of x and y satisfying this equation are 38 and 27. So, the priest started with 38 flowers, offered 27 flowers in each temple and returned with the an empty basket. Verify.

34. EAST-SOUTH ROUTES

There are in all 20 paths (including the three shown as specimen). All these are shown on the next page. How many did you get by trial?



35. AN UNKNOWN DIGIT

From any where in N, we may replace a block of six 1's by a block of six zeros, as this will only amount to subtracting from N some multiple of 13. Also if the zeros appear at the left or right end of N, they may as well be deleted. So, if we delete 24 digits on the left and 24 digits on the right in N, we will be actually deleting 4 blocks each of six 1's each on the left and right in N without affecting the propertyof divisibility by 13. But, we will be left with only a two digit number whose first digit is 1 and whose second digit is not known. If this two digit number is divisible by 13, then the unknown digit must be 3 which is the 26th digit of N.

36. WRONG LABELS

It is sufficient to draw one ball only from the box marked WB to reveal the contents completely of all the three boxes. For, suppose that the ball drawn is white, then obviously, the other ball must also be white as WB is a wrong label. It also follows then that two black balls must be found only in the box labelled WW and not the only remaining box as it is labelled BB. This box can only contain a white and a black ball. The procedure is similar if the first ball drawn from the WB labelled box was black.

37. DOUBLE AMOUNT

Let Arun have x rupees and Ashok y rupees. Then we have

$$x + 9 = 2\,(y - 9)$$

and $$2\,(x - 5) = y + 5$$

That is, $$x - 2y = -27$$

$$2x - y = 15$$

Multiply the second equation by 2 and subtract the first. We get $3x = 57$ or $x = 19$ and $y = 23$.

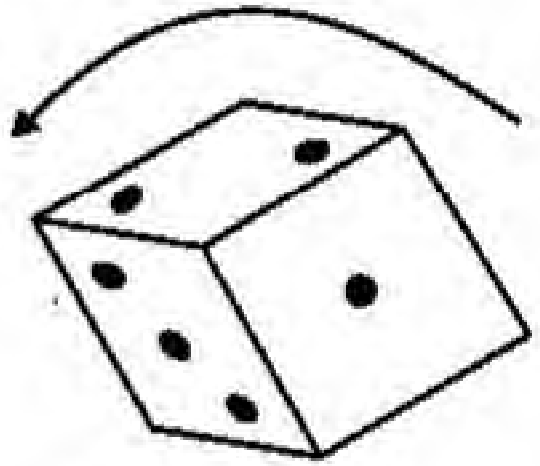
38. PILGRIM SADHU

Imagine a person X starts from the foot of Mt. Girnar at 8.00 a.m. and reaches the temples at 2.00 p.m. ascending with varying speeds which are exact imitations of the sadhu's journey, so that he would be with the sadhu all the while, if he did the journey on Monday. Actually let him do this journey on Saturday, so that X and sadhu both begin their journey at 8.00 a.m. on that day and both finish at 2.00 p.m., X ascending (like the sadhu on Monday) and the sadhu descending. Surely, X and the sadhu will meet at some point on the route. Obviously, this is the point whose existence was to be proved.

39. FARMER'S WILL

The judge added his cow to the farmer's 17 cows and started partitioning the total of 18 cows. Half of 18 is 9, so he gave 9 cows to the first son. One third of 18 is 6. So he gave 6 cows to the second son.One ninth of 18 is 2. So he gave 2 cows to the last son. On the whole, he gave 9 + 6 + 2 or 17 cows and retrieved his own cow. Thus he satisfied them all.

40. A DICE TRICK



If you hold a dice in your hand in such a way that the corner common to the three small numbered faces is facing towards you, you will see the order of these faces in the anticlockwise direction. The small numbered faces are the faces which contain one, two and three dots. In a normal dice, held in the

hand as described, these faces, when viewed in this order will show anticlockwise direction.

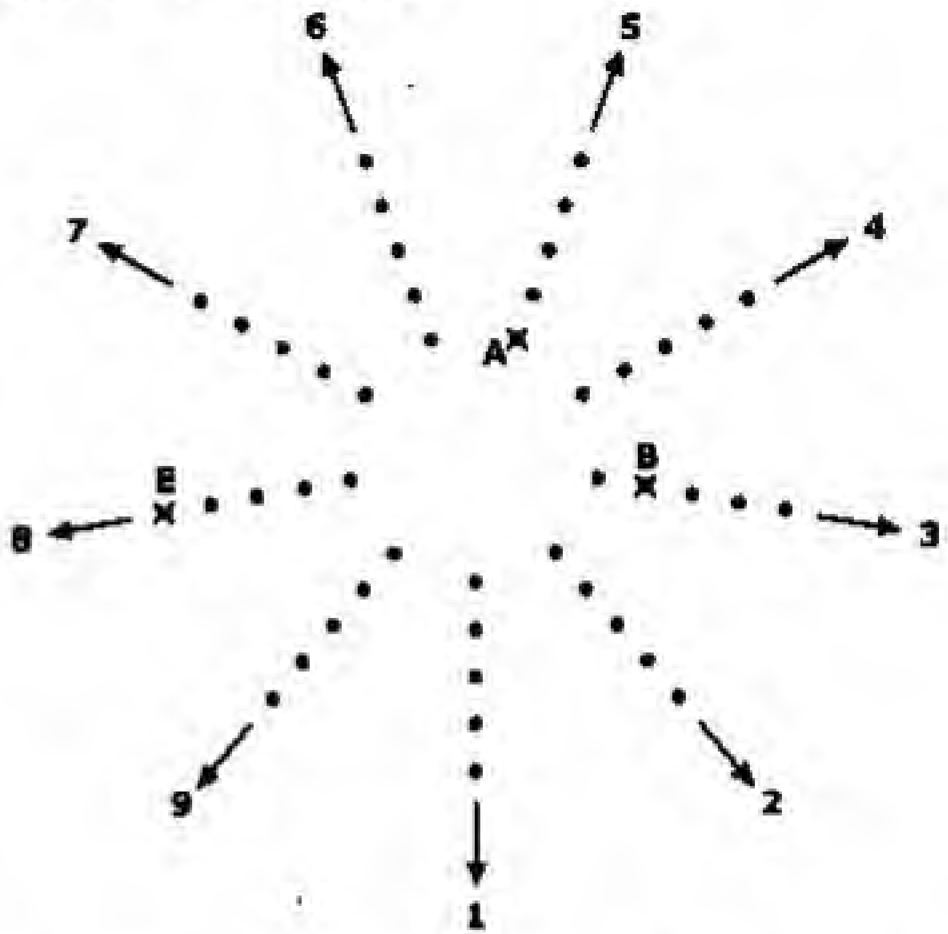
Go round the table on which the two dice are stacked one over the other with the top face covered. Four faces of each cube will be visible. Two of these visible faces will be small numbered ones. If you know the convention described above about the anticlock- wise direction, you can always use your imagination, and can determine whether the remaining small numbered face is at the bottom or top of the cube. It is enough if you find out one face. The other is got by subtracting the number from 7.

41. DIGITS IN A TRIANGLE

Left Scheme: Each digit is the difference of the two digits just above it.

Right Scheme : Each digit is the remainder got by dividing the two digit number above it by 7.

42. UNICURSAL DESIGN



The points referred to in the hints fall in sets of five in nine equally spaced directions as shown in the diagram here.

For convenience, the directions are designated by numbers 1 to 9. The set of five points in each direction may be named by the letters A,B,C,D and E with the number of the direction as a suffix. That is, all the nine points nearest to the centre are denoted by A; then the nine points next further away from the centre are denoted by B and so on; the farthest nine

points are E's. To explain this-by examples, the three points shown by crosses (X) are B_3, A_5 and E_8.

Now, to draw the design, you may start from any point say A_1, join this by a straight line to C_2, then go from C_2 to E_3, then to B_4, to D_5, and so on following the sequence below :

$A_1C_2E_3B_4D_5A_6C_7E_8B_9D_1A_2C_3E_4B_5D_6A_7C_8E_9B_1D_2A_3C_4E_5B_6D_7$
$A_8C_9E_1B_2D_3A_4C_5E_6B_7D_8A_9C_1E_2B_3D_4A_5C_6E_7B_8D_9A_1$.

43. AN AGE PROBLEM

The difference between Maya's and Dora's ages is obviously (?) 17 + 17 = 34 years. Therefore, Maya was 34 when Dora was born. Then Dora's father was 37. His marriage took place 5 years earlier, that is, when he was 32.

44. REVERSED DIGITS

In the Hints, we showed that A = 2 and E must be either 3 or 8. But as 4A = E, E can only be 8 and there is no carry over from 4B. Therefore, B = 0 or 1. As 4E = 32, the position of B in the product shows that B is not zero. So B = 1. With 3 carried over from 4E and having 1 in the place of B in the product, 4D ends in 8. Therefore, D = 7. Is it now difficult to see that C can only be 9 ? We have, then, 21978 x 4 = 87912.

45. PENTAGRAM NUMBERS

The sum of the ten numbers from 1 to 12, dropping 7 and 11 is 60 and since each number is taken twice, the total of all the twenty numbers is 120. As they are in five equal rows, the constant sum of each row is 24. Now, there are only fourteen sets each of four numbers as given below :

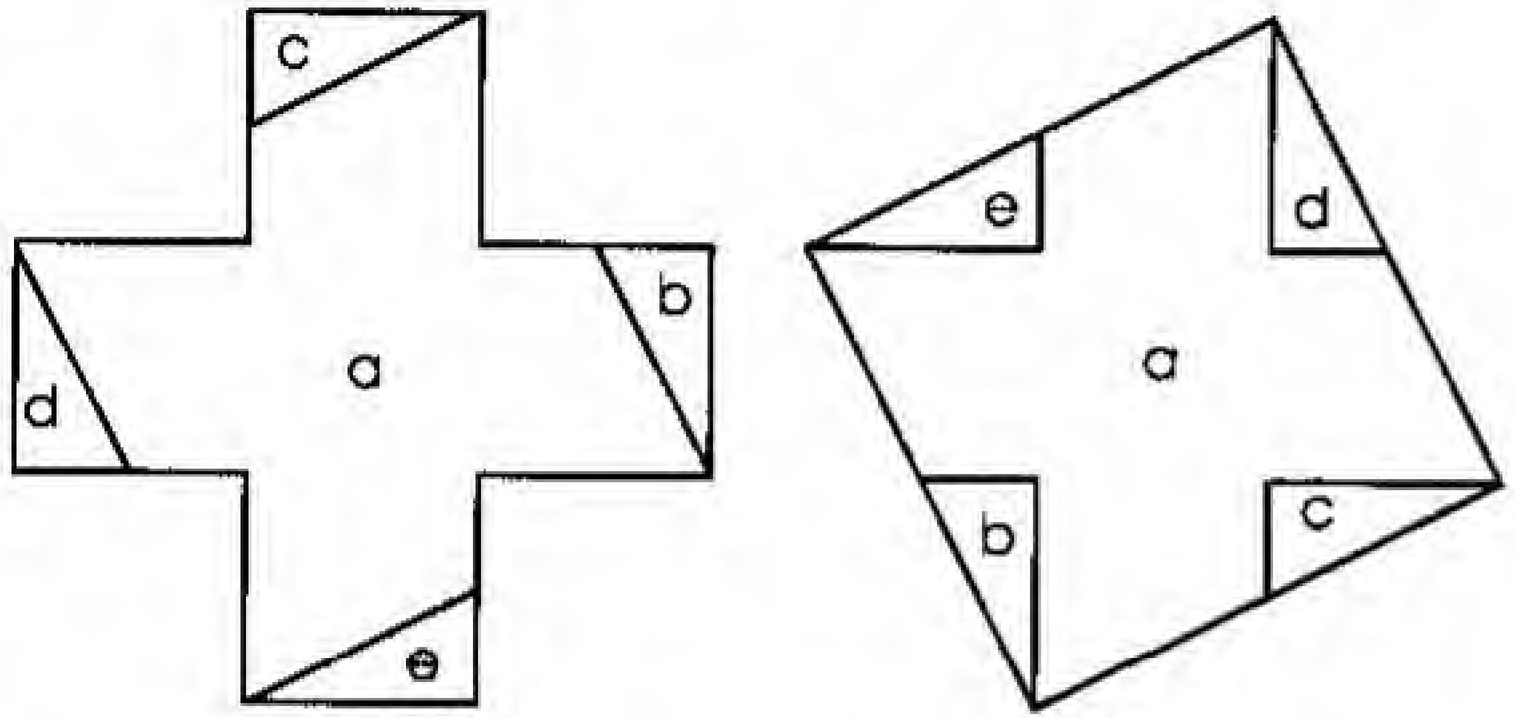
12	9	2	1	10	8	5	1
12	8	3	1*	10	8	4	2
12	6	5	1	10	6	5	3*
12	6	4	2*	9	8	6	1
12	5	4	3	9	8	5	2*
10	9	4	1*	9	8	4	3
10	9	3	2	9	6	5	4

The five rows marked with asterisk satisfy the condition (2). We repeat them here as our answer:

12	8	3	1
12	6	4	2
10	9	4	1
10	6	5	3
9	8	5	2

46. CROSS AND SQUARE

The solution is obvious from the figure shown.

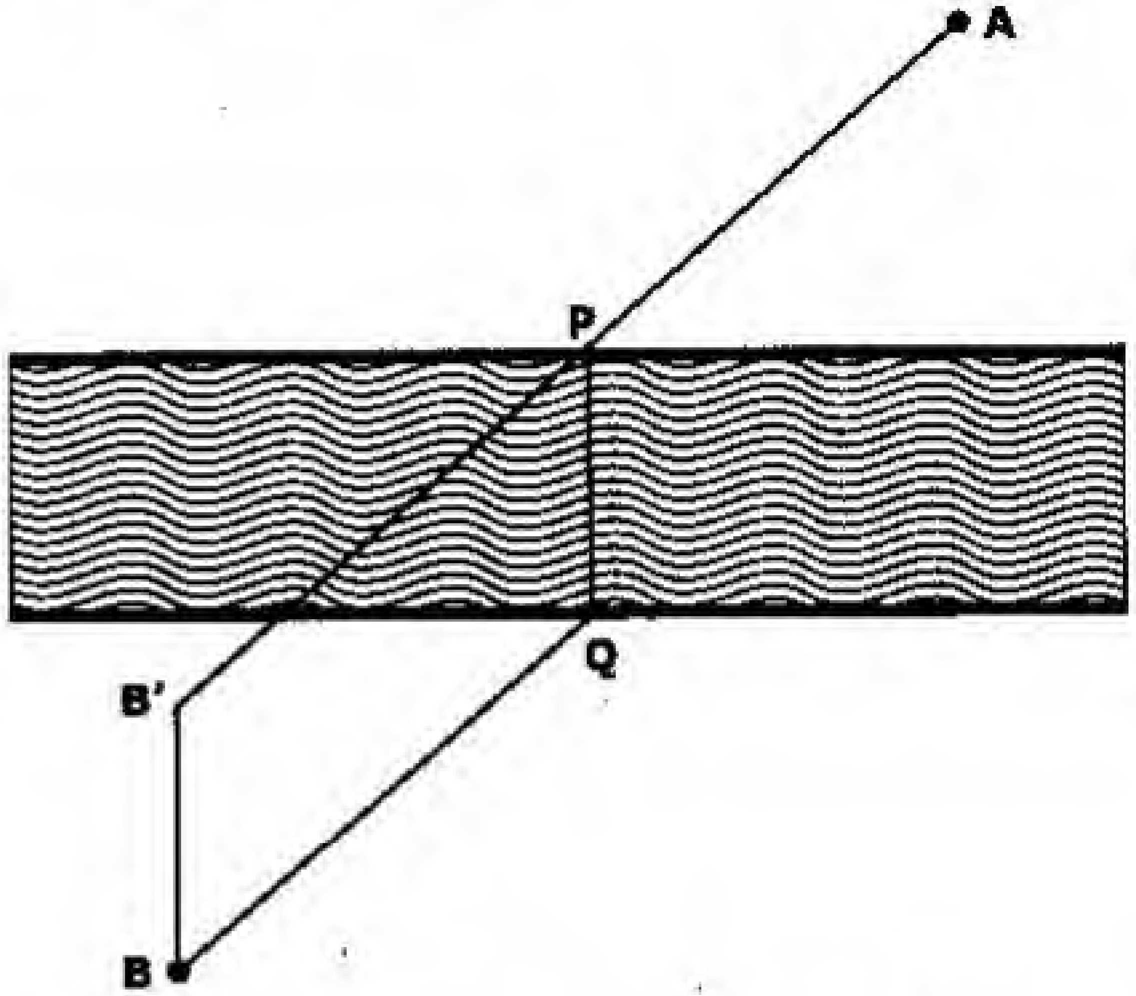


47. A COINS PROBLEM

From what is stated in the Problem and Hints, we have

$x+y+z=50$; $\frac{1}{2}x+y+5z=30$. Multiplying the second equation by 2 and subtracting the first from it, $y+9z=10$. As y and z are both whole numbers, the only possible values are $y=1$, $z=1$. Hence $x=48$.

48. A BRIDGE PROBLEM



Perform the following construction. From B draw a segment of line BB' perpendicular to and towards the banks, such that BB' = width of the river. Join AB' meeting the A side bank in a point P. Then draw PQ perpendicular to a bank and meeting the B-side bank in Q. Then PQ is the place where the bridge should be built.

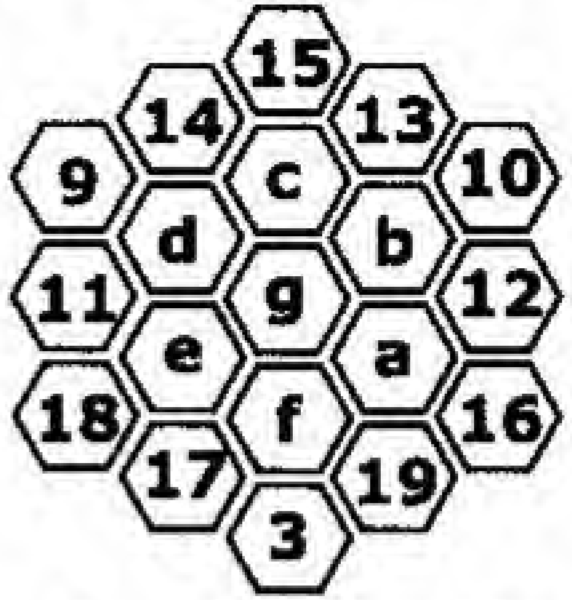
49. THE MISSING DIGIT

Add the digits your friend gives you. If the result is a number of more than one digit, add the digits of this number again. Continue this process until you get a single digit. Subtract this digit from 9. That will be the digit struck out by your friend. Example: suppose your friend started with the number 85697. He will subtract from this 8 + 5 + 6 + 9 + 7 = 35. He will get 85662. Let us say that he chooses to strike out the middle digit 6. He will tell you that the remaining digits are 8,5,6,2. When you get this, you add the digits of this and get 21. Again add 2 and 1, and get 3. Now 9 minus 3 is 6 and so you announce that 6 was the digit struck out and you will be correct.

50. WATER AND WINE

The correct answer is that the two amounts are equal. Because, both glasses were exactly half-full before the transfers of liquids and exactly half-full again after the transfers. So, whatever is lost in either glass must have gone in the other and vice versa. So, after the two transfers, the volume occupied by wine in the water glass must equal the volume occupied by water in the wine glass.

51. MAGIC HEXAGON



The sum of all the numbers from 1 to 19 is 190. This is equally distributed among five parallel (say vertical) lines. So, the constant sum for each line must be 38. Thus, the entries in the remaining six border cells are as shown in the figure. Now, suppose the entries in the inner seven cells are *a, b, c, d, e, f, g* *as* shown. Using the fact that the constant sum is 38, we have, obviously $a + b = 6$. This is possible only if $(a, b) = (5, 1)$ or $(1, 5)$ or $(4, 2)$ or $(2, 4)$. Only one of these is true and to find this, we use every time the constant sum property. $a = 5$ implies $f = 4$ which implies $e = 4$ and this is impossible, $a = 1$ implies $f = 8$ which implies $e = 0$ and this is impossible. Next, $a = 4$ implies $f = 5$ which implies $e = 3$ and this is impossible as 3 has already appeared. So, it is proved that $a = 2$, $b = 4$. This implies that $c = 8$, $d = 6$; $e = 1$ and $f = 7$ and the central figure $g = 5$.

The complete solution is shown in the figure.

52. REMAINING REMAINDER

First, you should know the relationship among the three remainders, referred to in the Hints. Suppose the remainders are x, y, z in this order. If you are given x and y, you must find z. For this, divide $4x - 3y$ by 13 and the remainder will be z. (If $4x - 3y$ is negative, then add to it 13 or a suitable multiple of 13 to get a positive number less than 13). In the example, given in the problem, suppose you are given 5 and 12. Then, $4 \times 5 - 3 \times 12 = -16$. Adding 2×13 or 26 to this, you get 10 as the third remainder. But, if you are given 12 and 10, you note that $4 \times 12 - 3 \times 10 = 18$. Dividing this by 13, you get 5 as the remainder, which is the required answer. Also $4 \times 10 - 3 \times 5 = 25 = 13 + 12$ and you say that the third remainder is 12.

Take another example. Let your friend start with the three digits 2, 8, 9. From this, the three two-digit numbers will be 28, 89, 92 in this order. The remainders after dividing by 13 are 2, 11, 1. If he tells you that two of the remainders are 11 and 1, how will you find the remainder 2 ? This is how : $4 \times 11 - 3 \times 1 = 41 = 3 \times 13 + 2$. Or from 1 and 2, you get $4 \times 1 - 3 \times 2 = -2 = -13 + 11$ and so 11 is the answer. Also $4 \times 2 - 3 \times 11 = -25 = -2 \times 13 + 1$ and so 1 is the answer.

Use your knowledge of mathematics to find the reason why this always works, if you can't, then see the comments for a proof.

53. A MULTIPLE OF 11

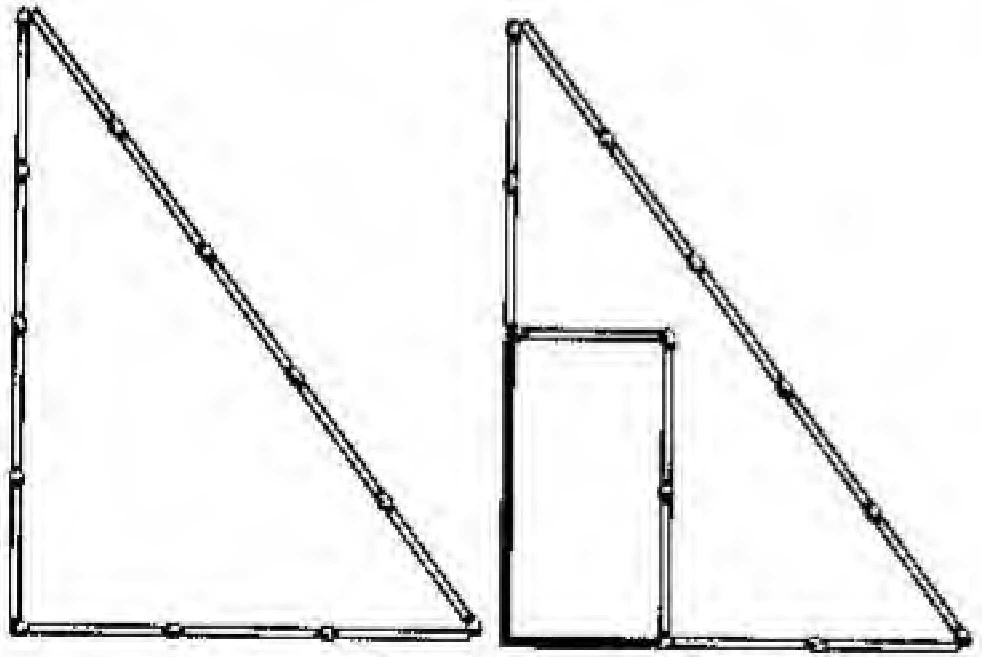
The biggest number of nine different digits is, obviously, 987654321. But this is not divisible by 11, as the difference of the sums of odd-placed digits and even-placed digits (which we may call the test-difference for convenience) is 5, not 11. We should try to shuffle some last few digits until the test- difference becomes 11. Thereby the number will remain as large as possible as we do not tamper with the left side digits. Interchange of 2 and 1 does not help. Any permutation of the last three digits also does not help. We should, therefore, try some permutation of the last four digits 4,3,2,1. Now the test-difference in 98765 is 7. This falls short of 11 by 4 which must come from some arrangement of 4,3, 2,1. This is easily seen to be 2413. So, the required biggest multiple of 11 is 987652413.

54. A CHAIN PROBLEM

The correct answer is Rs. 9.00. What he has to do is to break open all the three links in one bit, put them in the three gaps between the other bits and then close them up. This is a test of one's ingenuity.

55. TWELVE MATCHES

As $3^2 + 4^2 = 5^2$ and $3 + 4 + 5 = 12$, a triangle whose sides are 3, 4 and 5 matches, is a right angled triangle. The area of this triangle is equal to half the base x height, that is, six match squares. Now rearrange three of the matches as shown in the figure, so as to deprive the total area by two match squares. So the new enclosed area will be equal to four match squares.



56. HOW MANY SQUARES

The sides of a 2 x 2 square any where in the figure will (produced, if necessary) cut out a 2-unit segment in the length and a 2-unit segment in the width of the rectangle. Conversely, any 2-unit segment in the length and any 2 unit segment in the width will determine a 2 x 2 square. But, there are nine 2-unit segments in the length and six 2-unit segments in the width. So, the total number of 2 x 2 squares is 9 x 6 or 54. Similarly, the number of 3-unit segments in the length is 8 and in the width 5. So, there are 8 x 5 or 40 3 x 3 squares and so on. Hence the total number of squares of all sizes is $10\cdot7 + 9\cdot6 + 8\cdot5 + 7\cdot4 + 6\cdot3 + 5\cdot2 + 4\cdot1 = 70 + 54 + 40 + 28 + 18 + 10 + 4 = 224$.

57. WEDDING INVITEES

Let the number of bicycles be x, autorikshas y and cars z. Then, we have the following three equations :

$$x + y + z = 50 \quad \text{(i)}$$

$$2x + 3y + 4z = 139 \quad \text{(ii)}$$

$$x + 2y + 5z = 109 \quad \text{(iii)}$$

Adding (i) and (iii) and subtracting(ii), we get $2z = 20$ or $z = 10$. Subtracting (i) from (iii), $y + 4z = 59$. Therefore $y = 19$. Therefore autorikshas were 19, cars were 10 and bicycles were 21.

58. BUYING COCOANUTS

The bigger cocoanut is really cheaper. Because, the volumeof the bigger: the volume of the smaller $= \left(1\frac{1}{2}\right)^3 = \left(\frac{3}{2}\right)^3$ $= \frac{27}{8} = 3\frac{3}{8}$. Since the quantity of kernel is proportional to the volume, the bigger cocoanut will give nearly three and a half times more kernel than a smaller one, while the cost is only double. So, the bigger cocoanut is cheaper. It is, of course, assumed that all cocoanuts big or small have similar shapes, if not nearly spherical and that there is no other difference, as in the quality or taste of the kernel.

59. PROFIT AND LOSS

Where there was a profit of 10%, if the selling price was Rs. 110, the cost price would be Rs. 100. So, as the selling price is 1584, the cost price was $\frac{1584 \times 100}{110} = 1440$. Where there was a loss of 10%, if the selling price was Rs. 90, the cost price would be Rs. 100. But as the selling price is 1584 the cost was $\frac{1584 \times 100}{90} = 1760$. So, the total cost price was Rs. 3200 while the total selling price was Rs. 3168. Thus, there was a loss of Rs. 32 which is 1% of the cost price.

60. A LIE

The number of marbles Shantilal brought home was a perfect square. Say, it is x^2, where x is a natural number. The number of marbles he left in the tray would be $\frac{1}{2}x^2$. If n is the number of marbles Ashok took, then Asha must have taken $\frac{1}{2}n$ marbles. So, we must have $\frac{1}{2}x^2 = \frac{3n}{2} + 1$ or $x^2 = 3n + 2$. But, this is impossible, because no perfect square can be of the form $3n + 2$. For, n must be of the form $3r$, $3r + 1$ or $3r + 2$. So, x^2 is of the form $9r^2$ or $9r^2 + 6r + 1$ or $9r^2 + 12r + 4$ which is of the form $3m$ or $3m + 1$. Hence $3n + 2$ cannot be the form of a perfect square. So, Asha must have lied.

61. HALF DISTANCE - HALF TIME

Let x be the distance from home to school, u, running speed, v, walking speed, T, time taken in the first case and t the time taken in the second case. Then, we have

$$\frac{\frac{1}{2}x}{u} + \frac{\frac{1}{2}x}{v} = T$$

$$\frac{1}{2}tu + \frac{1}{2}tv = x$$

$$\therefore t = \frac{2x}{u+v} \text{ and } T = \frac{(u+v)x}{2uv} \therefore \frac{T}{t} = \frac{(u+v)^2}{4uv}$$

But, as we know that $(u+v)^2 > 4uv$, we have $T > t$. So, the second alternative will take Raju quicker to the school.

62. PERFECT SQUARE

The largest number of five digits in base seven is 66666. In decimals, this is 16806. The largest perfect square less than this is 129^2. Now convert $129^2 = 16641$ into its base seven equivalent.

The answer is 66342

```
      1
      7
     49
    343
   2401
   2801
  x   6
  16806
```

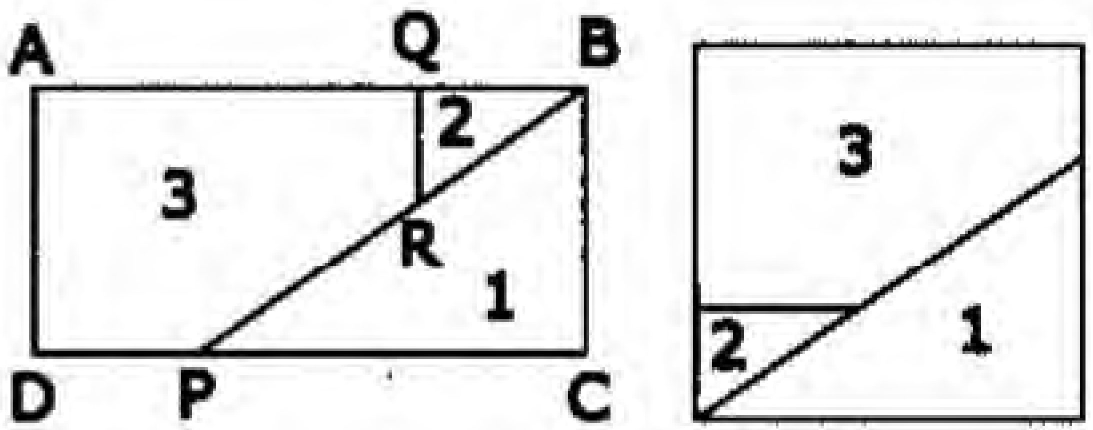
```
  1) | 1,68,06 (129
     | 1
 22  | 0 68
     |   44
 249 |   2406
     |   2241
```

```
7 | 16641 |
7 |  2377 | 2
7 |   339 | 4
7 |    48 | 3
  |     6 | 6
```


63. RECTANGLE - SQUARE

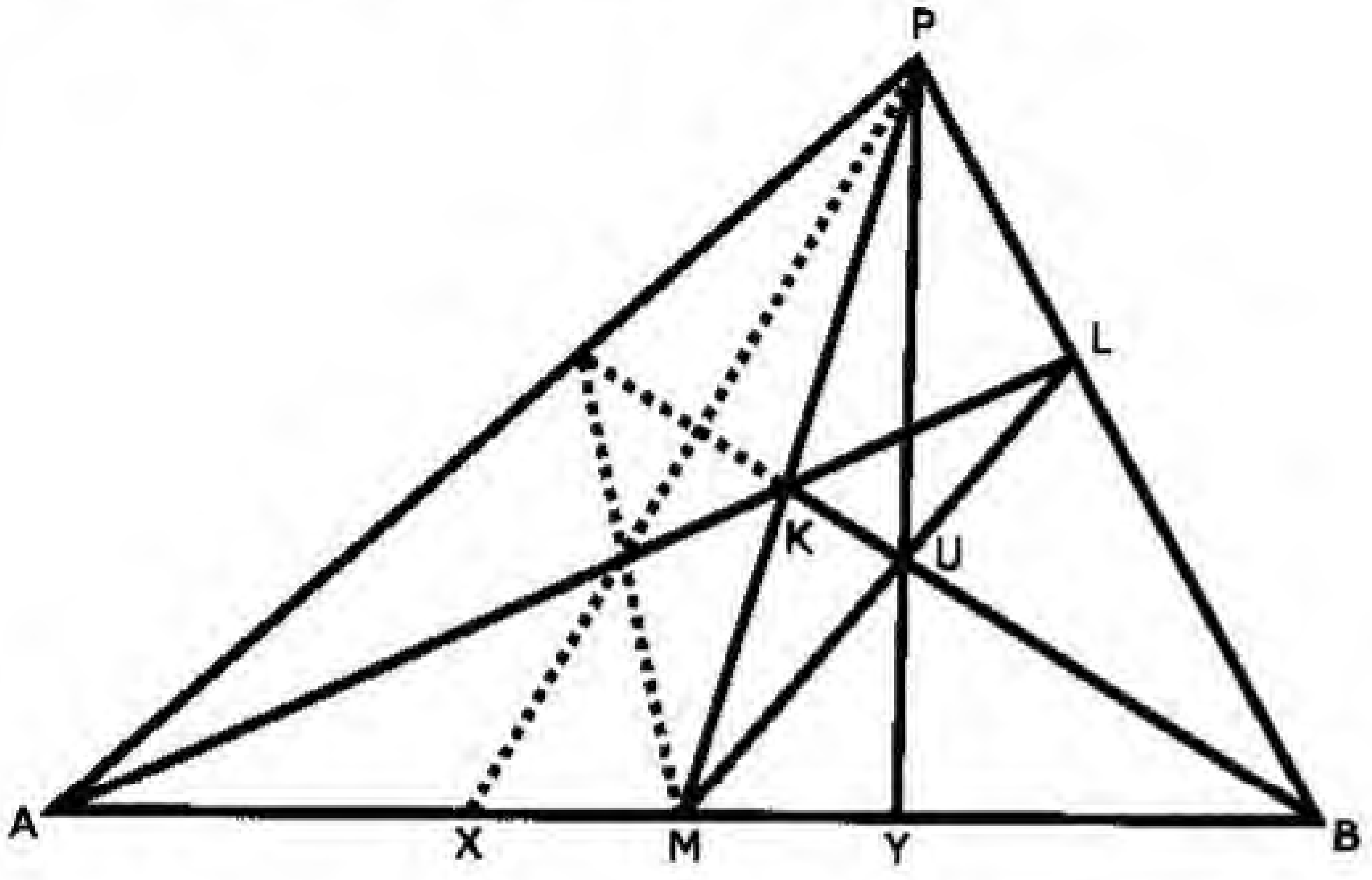
As suggested in the Hints, a practical way of getting a side of the square equal in area to a given rectangle is to measure correctly the length a and width b of the rectangle, multiply and take the square-root. We have $\sqrt{ab} = x$.

The result will be approximate. A theoretical construction is described in the comment section.



Let ABCD be the rectangular sheet of paper given. Along the length $\overline{CD}$, cut off CP = x and along AB, cut off AQ = x. Join $\overline{PB}$ and let the perpendicular to $\overline{AB}$ at Q meet $\overline{PB}$ in R. Then cut the paper with scissors along PB and QR. You will get the three pieces which can be rearranged to make a square as shown in the second figure above.

64. RULER CONSTRUCTION



Take any point P not on $\overline{AB}$. Join $\overline{PA}$, $\overline{PM}$, $\overline{PB}$. Draw any line through A meeting $\overline{PM}$ in K and $\overline{PB}$ in L. Join $\overline{BK}$ and $\overline{ML}$ meeting each other in U. Join $\overline{PU}$ and produce it to meet AB in Y, which is one of the points of trisection wanted. The construction for the other point X is similar and is shown in the figure.

65. COLOUR CUBES

The answer is 30

Proof : Let the answer be n. Imagine n cubes all differently painted in the manner described and lying on a table in front of you. Let the colours, for definiteness, be black, white, red, yellow, blue and green. First choose one colour, say, black. Since every cube has each colour on one of its faces, find the black colour on each of the n cubes and turn it, if necessary, so that the black colour is at the bottom (touching the table and so out of sight). You will find the remaining five colours on all the various top faces of the n cubes. There is no reason why any one of these five colours should occur more or less often than any other. So, n must be divisible by 5, and there will be $\frac{n}{5}$ cubes having one and the same colour on the top faces. Choose any one colour say, white and keep the $\frac{n}{5}$ cubes with white tops and remove the rest $\frac{4n}{5}$ cubes from the table. What you have on the table, now, are $\frac{n}{5}$ cubes, all with black bottom and white top and the remaining four colours distributed on the vertical faces. Let these cubes stand in a row and you turn each about a vertical axis so that a third colour, say, red on every cube comes in front, facing you. Now, look at the rear faces.

These will have the remaining three colours, yellow, blue and green variously distributed. Again, there is no reason why any one of them should occur more or less often than any other. So $\frac{n}{5}$ must be divisible by 3. If we choose yellow, rejecting the other two colours, we know, now, that there will be $\frac{n}{15}$ cubes, in all of which we have bottom black, top white, front red, and rear yellow. There remain only two colours blue and green for the two faces, left and right. Obviously, there will be only two cubes with left face blue, right face green and left face green, right face blue. Thus $\frac{n}{15} = 2$ and, therefore, $n = 30$.

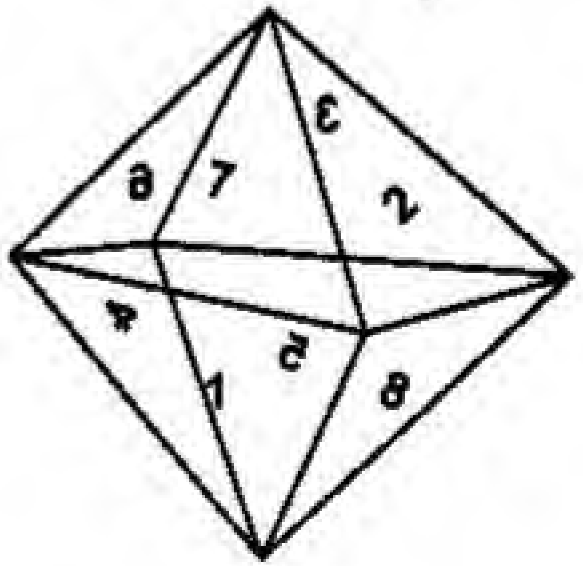
66. MAGIC OCTAHEDRON

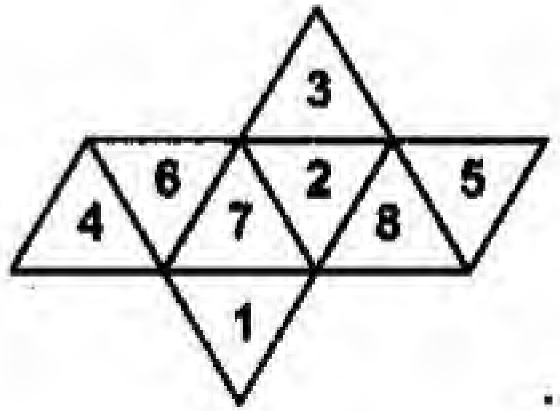
Note that the sum of all the numbers from 1 to 8 is 36. If you hold the octahedron in any manner in front of you, you will always be able to see just four out of the eight faces and the numbers on these faces must give the constant sum. The remaining four faces on the other side will have numbers also totalling to the same constant. Hence, the constant sum is 18. Now, using numbers, 1 to 8, there are only eight ways of expressing 18 as a sum of four different numbers.

They are as follows :

$8+7+2+1$ (a)	$7+6+4+1$ (e)
$8+5+4+1$ (b)	$7+6+3+2$ (f)
$8+5+3+2$ (c)	$6+5+4+3$ (g)
$8+6+3+1$ (d)	$7+5+4+2$ (h)

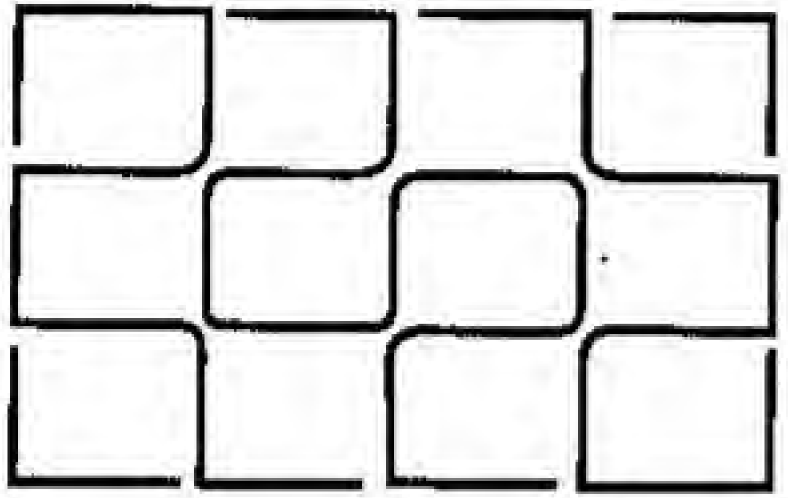
The numbers on faces corresponding to each of the three pairs of opposite corners can be chosen from the above as (a, g), (b, f), (c, e). Also two numbers will be common to the two sets of four numbers associated with any two adjacent corners. Noting these properties, it should be easy to write the numbers on the eight faces appropriately. The numbers appearing on the net will be as shown here.





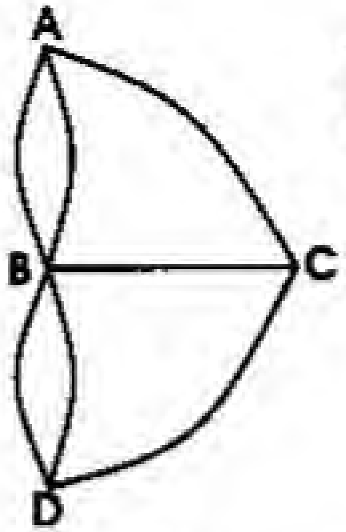
67. RECTANGULAR MESH

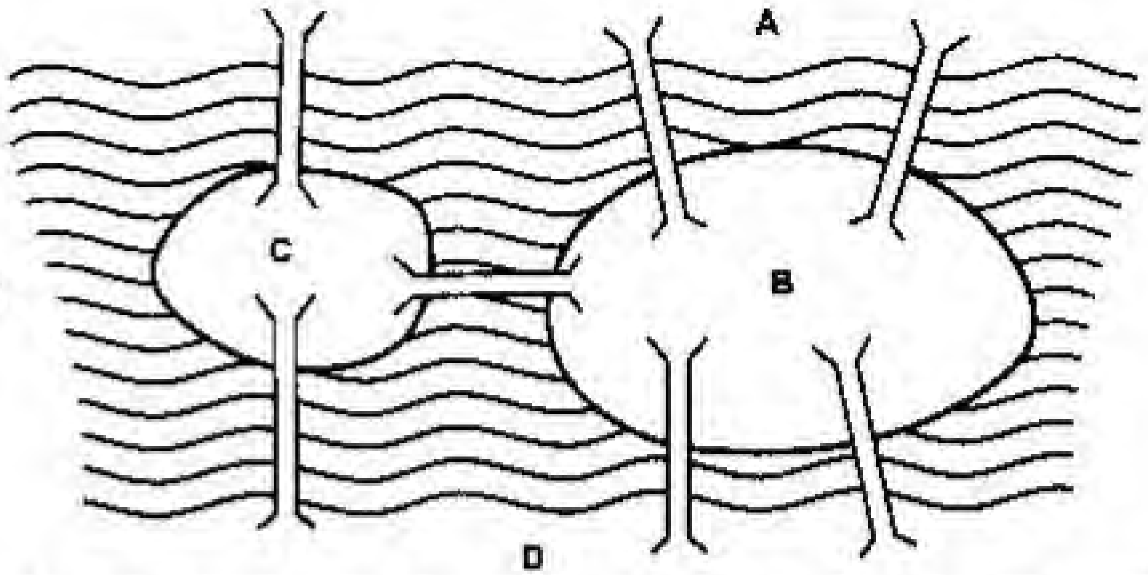
There are many ways of doing this. One of the ways is shown here.



68. KÖNIGSBERG BRIDGES

There are four land pieces A, B, C, D. Represent them by four points A, B, C, D (using the same letters). Each of the seven bridges may be represented by a line joining the points which correspond to the land pieces joined by the bridges. Thus we get the design shown on the next page.





The problem of walking over each bridge only once is, obviously, equivalent to tracing the representative design unicursally, i.e., without lifting the pencil off the paper or retracing a line. But, referring to the comments on problem 67, we know that this is impossible, as all the four points are odd nodes. So, we will have to lift the pencil once to completely draw the design.

69. A PARADOX

We cannot say whether the statement (1) is true or false in a paradox. For, we know that statement (4) is false. Statement (6) is also false, as the two lines may not be in the same plane. All the other statements are true. So, if you take (1) to be also false, then there would be three false statements and so what is stated in (1) will be true. On the other hand if

(1) is true, there are only two false statements in the list and what is stated in (1) is false. Thus, if the statement (1) is false only if it is true, and true only if it is false ! This is inconsistent.

70. DIGIT PAIRS

Here is a proof that there is no solution to this problem. If possible, suppose the digits 1, 2, 3, 4 and 5 each repeated are written in a line, in ten numbered places, satisfying the condition.

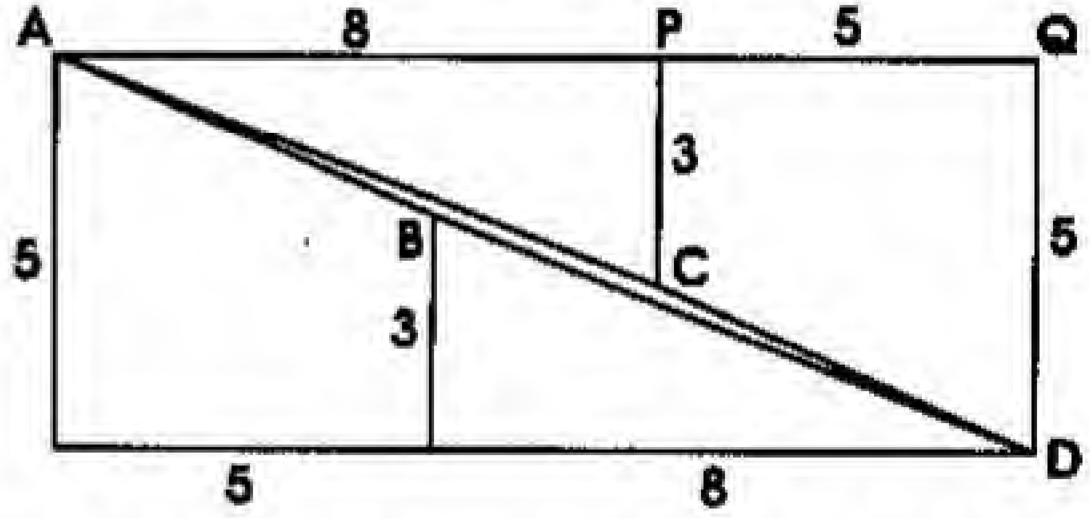
Suppose the first occurence of 1 is in place a, that of 2 is in place b, 3 in c, 4 in d, and 5 in e. Obviously, the second occurrence of 1, 2, 3, 4 and 5 will be in place $a+2$, $b+3$, $c+4$, $d+5$ and $e+6$ respectively. The sum of all these place numbers is clearly $1+2+3...9+10=55$. The numbers are same as a, b, c, d, e, $a+2$, $b+3$, $c+4$, $d+5$, and $e+6$ in different order. So, we have $55=2(a+b+c+d+e)+20$.

This is impossible, for, 55 is odd, while the right hand side is even. Therefore, there is no solution.

71. 63 = 64 = 65

We take the first figure, in which there are 64 unit squares as correct. Then, the other two figures are both incorrect. For,

the four points A, B, C, D which appear as collinear are really not collinear. Because, if they are in a line, then the two triangles APC and AQD will be similar. So, we should have $\frac{AP}{PC}=\frac{AQ}{QD}$ i.e., $\frac{8}{3}=\frac{13}{5}$, or 40 = 39, which is absurd. As the error is 1 in 40 or only $2\frac{1}{2}$ %, the eye is not able to perceive it and hence the illusion. In a correct figure, A,B,C,D will be the vertices of an elongated parallelogram whose area is as small as one square unit embedded in a space of 65 square units, as shown here.



Can you not explain on similar lines how the third figure is also wrong?

72. CONSTANT SUM

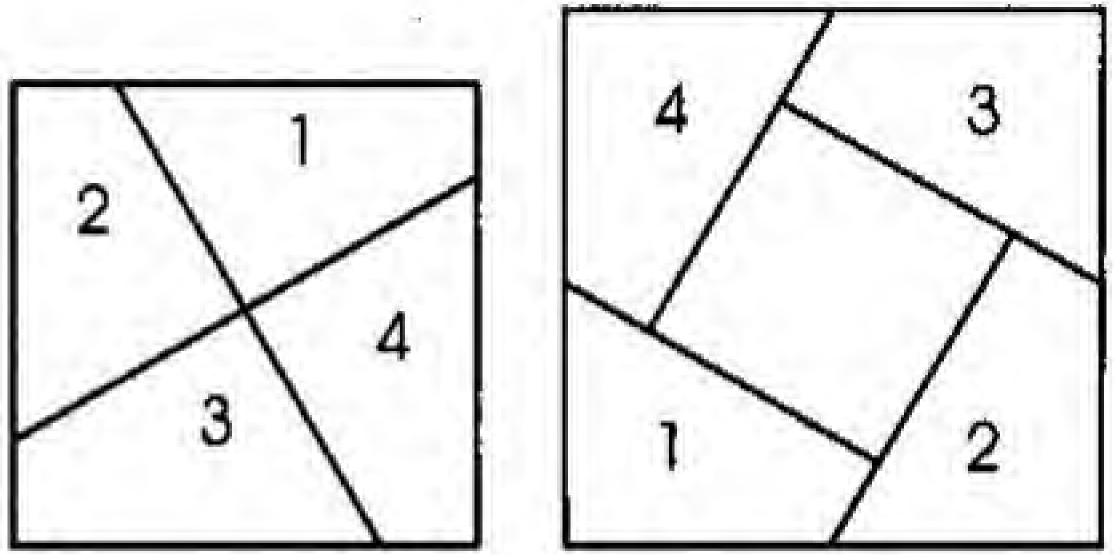
At the top of the 25 cell square, introduce a row of five numbers and to the left of the square, a column of another set of five numbers as shown here.

	4	1	13	7	10
8	12	9	21	15	18
4	8	5	17	11	14
19	23	20	32	26	29
0	4	1	13	7	10
15	19	16	28	22	25

A number in every cell is the sum of the two numbers, one which lies at the top of the column in which the chosen number is and the number at the extreme left in the row in which the chosen number lies. Thus, if five cells are chosen so that no two of them lie in the same row or column and the number in each cell is replaced by the two numbers in the top row and left column introduced by us, then we will get all the ten numbers introduced by us whose sum (81, here) is, of course, independent of the choice of cells.

73. A SQUARE HOLE IN A SQUARE

You have to rearrange the four pieces as shown in the figure here.



74. FOUR 4's

$$13 = \frac{4-4}{.4} + 4$$

$$17 = 4 \times 4 + \frac{4}{4}$$

$$52 = 44 + 4 + 4$$

$$68 = \frac{4^4}{4} + 4$$

75. SEVENTEEN DOMINOES

1	2	3	4	5	6
12	11	10	9	8	7
13	14	15	16	17	18
24	23	22	21	20	19
25	26	27	28	29	30
36	35	34	33	32	31

Number the cells from 1 to 36, as shown. Obviously, a domino placed anywhere on the square vertically or horizontally will always cover two numbers of different parity, i.e., one odd and one even and never two numbers of same parity. So, seventeen dominoes should cover 17 odd and 17 even numbers. Hence, if the two blocked cells contain numbers of different parity, then it will be possible to fit in the 17 dominoes. This is the case in the first figure. For, the numbers of the blocked cells are 8 and 25 which are even and odd respectively. Here, it is possible to fit in the dominoes. But, if the blocked cells contain numbers of same parity, say, both odd, there will be available 18 even numbers and 16 odd numbers for covering with dominoes. So only 16 dominoes can be laid on the square and the two remaining cells are both even numbered and so cannot be covered by a domino.

This is the case in the second figure in which the numbers blocked are 15 and 33, both odd. Here, it is impossible to fit in the dominoes.

76. EIGHTEEN DOMINOES

It is impossible to find such an arrangement.

Proof: There are just 10 such free lines (going from one edge to the opposite edge) that are to be considered, five vertical and five horizontal. Each such line divides the square into two parts in each of which there are an even number of squares. Consider any one of these lines. If it is obstructed by a domino, this domino will contribute one square to each part described above. In that case, this cannot be the only obstructing domino, as that will render the number of squares in each part odd. Hence, if an obstruction takes place on any line, it will be by at least two dominoes. We want that every one of the ten lines should be obstructed. So we will require 20 dominoes for this, where as we have only 18. So, there must be at least one of the ten lines free to pass unobstructed.

77. FIVE BY FIVE COLOUR SQUARE

The solution is as shown in the figure.

A	C	E	B	D
B	D	A	C	E
C	E	B	D	A
D	A	C	E	B
E	B	D	A	C

78. ODD EVEN ALPHAMATICS

Let us rewrite the multiplication as follows:

$$\begin{array}{r} E_1\,E_2\,O_1 \\ E_3\,O_2 \\ \hline O_3\,O_4\,O_5 \\ O_6\,E_4\,E_5 \\ \hline O_7\,E_6\,O_8\,O_9\,O_5 \end{array}$$

O_7 is only a "carry-forward" digit and so $O_7 = 1$. In the previous column, $O_6 = 9$ and $E_6 = 0$ for, otherwise, there will be no "carry-forward". As $O_6 = 9$, we must have $E_3 \times E_1 = 8$. Now, there are only three digits in the third line. So, $E_1 \neq 4$, for, otherwise O_2 ($\neq 1$) must be at least 3, the third line will have four digits. So, $E_1 = 2$ and $E_3 = 4$. For a similar reason, as $E_1 = 2$, O_2 is not greater than 3 and so $O_2 = 3$. To account for O_4 which comes from $O_2 \times E_2$, there must be only 1 carried over in $O_2 \times O_1$. So, $O_1 = 5$.

As $O_6 = 9 = E_1 \times E_3 + 1$ with $E_3 = 4$, 'carry over' of 1 in $E_3 \times E_2$ is possible only if $E_2 = 4$ or 2. But $E_2 = 2$ will imply $O_2 \times E_1 = O_3$ which is absurd. So, $E_2 = 4$. Thus, the unique solution is,

$$\begin{array}{r} 245 \\ 43 \\ \hline 735 \\ 980 \\ \hline 10535 \end{array}$$

79. TWO FRIENDS

Let a point A represent Krishnan's house, B Rajesh's house and P the point where they crossed each other. Let u meters per minute be Krishnan's speed of walking and v Rajesh's speed.

Then, obviously, $AP = 8v$ and $PB = 18u$.

$$AB = 8v + 18u$$

Time taken by Krishnan to walk the distance AP

$$= \frac{AP}{u} = \frac{8v}{u}$$

Time taken by Rajesh to walk the distance BP

$$= \frac{BP}{v} = \frac{18\,u}{v}$$

$$\therefore \frac{8v}{u} = \frac{18u}{v} \qquad \therefore \frac{v^2}{u^2} = \frac{18}{8} = \frac{9}{4} \qquad \therefore \frac{v}{u} = \frac{3}{2}$$

Total time taken by Rajesh $= \frac{AB}{v} = \frac{8v + 18u}{v}$

$$= 8 + 18 \times \frac{2}{3} = 20 \text{ minutes}$$

$\therefore$ Rajesh was at Krishnan's house at 9.20. Similarly, Krishnan took $\frac{AB}{u}$ minutes $= \frac{8v + 18u}{u} = 8 \times \frac{3}{2} + 18 = 30$ minutes. So, Krishnan was at Rajesh' house at 9.30.

80. PAPER FOLDING

Before folding, let the length of the paperstrip AB be l. Then, the distance of the first crease from A is $a + \frac{l - a}{2}$, and the distance of the second crease from B $= b + \frac{l - b}{2}$. If the creases are at P and Q, we want PQ.

$$PQ = AP + BQ - AB = \left(a + \frac{l-a}{2}\right) + \left(b + \frac{l-b}{2}\right) - l$$

$$= \frac{a+b}{2}$$

81. TRIANGULAR TIER

The size of a triangle is measured by the number units in a side (or base). There are two types of triangles, erect (Δ) and inverted (∇). Let us count each type and add to get the answer.

Erect triangles :

on unit side there are	1 + 2 + ... + 10	= 55
on 2-unit side there are	1 + 2 + ... + 9	= 45
on 3-unit side there are	1 + 2 + ... + 8	= 36
....		
....		
on 10-unit side there is	1	= 1

This totai is $55+45+36+28+21+15+10+6+3+1=220$

Inverted triangles :

on unit side there are	$1+2+3+...+9$	=	45
on 2-unit side there are	$1+2+...7$	=	28
on 3-unit side there are	$1+2+...+5$	=	15
on 4-unit side there are	$1+2+3$	=	6
on 5-unit side there is	1	=	1
Total		=	95

Hence, the total number of triangles of all sizes is $220+95=$ **315**.

82. ANAND'S AGE

Let Abhay's present age be n months. So Anand's present age is n^2 months. It is given that $n^2+51=m(n+51)$ where m is a natural number. Now, $n+51$ is a factor of n^2+51. $n+51$ is also a factor of n^2+51n. Hence, subtracting, $n+51$ is a factor of $51n-51$. $n+51$ is again a factor of $51n+51^2$. Subtracting, $n+51$ is a factor of 51^2+51.

Now, $51^2+51=51\times 52=2\times 2\times 3\times 13\times 17$

Consider all the divisors of this product and equate them one by one to $n+51$. Reject those which lead to trivial or absurd values, $n=1$, $n^2<720$ and $n^2>1200$, we get the only value $n=27$ giving $n^2=729$. Thus Anand retired 9 months ago.

83. 1 TO 80

For performing this difficult trick, you have to memorise some points. They are as follows.

Out of the eight sides of the four cards, only one side of one card has been used for the following special numbers : 1, 3, 9, 27, 41, 43, 49, 67.

The card and its side is thus identified by one of these numbers.

These numbers have 'duplicate values' as shown here :

For		Its duplicate	For		Its duplicate
1	→	–1	41	→	1
3	→	–3	43	→	3
9	→	–9	49	→	9
27	→	–27	67	→	27

After the subject has placed the cards on the table so that his

chosen number is on the underside of those cards, you have a look on the reverse sides of those cards, visible to you, identify them by the special numbers, find their duplicate values and add those values. If the sum is positive, you may announce that as the chosen number. If the answer is negative, remove the negative sign and add 40. You may announce that as the chosen number.

This will always work and surprise the subject. Some explanation can be found in the comments.

Take an example. Suppose the subject thinks of number 23. On the table, he will place the first, second and fourth cards with the right, right and left sides respectively as undersides. (The third card will not be shown at all). The numbers you will identify will be 1, 3, 67. Their duplicate values are – 1, – 3, 27. Adding these you get 23.

In another experiment, if the subject places two cards and you identify the sides 27 and 43, you should get – 27 + 3 = –24. The chosen number will then be 24 + 40 = 64.

84. 2000 AD

$$2000 = \frac{(2 \times 2)^2}{.2 \times .2 \times .2}$$

$$= \left(\frac{3}{.3}\right)^3 + \left(\frac{3}{.3}\right)^3 \qquad = \frac{(5+5)^5 \times .5}{5 \times 5}$$

$$= \sqrt{\left(\frac{7}{.7}\right)^7} \times \frac{\sqrt{.7}+\sqrt{.7}}{\sqrt{7}} \qquad = \left(\sqrt{\frac{8}{.8}}\right)^8 \times \frac{.8+.8}{8}$$

$$2000 = \sqrt{9} \times 8 - 7 + 654 \times 3 + 21$$

85. LAST THREE DIGITS

We write down schematically the actual multiplication showing the unknown digits by letters. The following values of the letters (each forced) are found easily step by step in the order shown. To see this clearly, the reader should at once replace each letter as soon as its value is found.

$f = 7, c = 3, e = 8, d = 1, h = 6, b = 4, g = 1, i = 8, a = 2$

The last three digits of the other factor are, therefore 243

```
   . . . 7 2 9
 x . . . a b c
 -------------
   . . . d e f
 . . . . g h
 . . . . i
 -------------
       1 4 7
```

86. BISECTION TIME

Note that round the rim of the face of a clock there are 60 divisions. Each division corresponds to 6° of angle, traversed by the minute hand in one minute and by the hour hand in 12 minutes.

Let the first bisection take place at x minutes past 8.

After crossing the vertical, the minute hand traversed $x-30$ divisions. $\therefore$ the hour hand is $2x-60$ divisions away from the vertical and so traversed $2x-70$ divisions after 8.

$$\therefore 12(2x-70)=x$$

$$\therefore x=\frac{840}{23}$$

Suppose the second bisection takes place at y minutes past 8. Now the minute hand is $60-y$ divisions from the upward vertical and the hour hand, will, therefore, be $120-2y$ divisions from the upward vertical. So, the hour hand has traversed $20-(120-2y)=2y-100$ divisions.

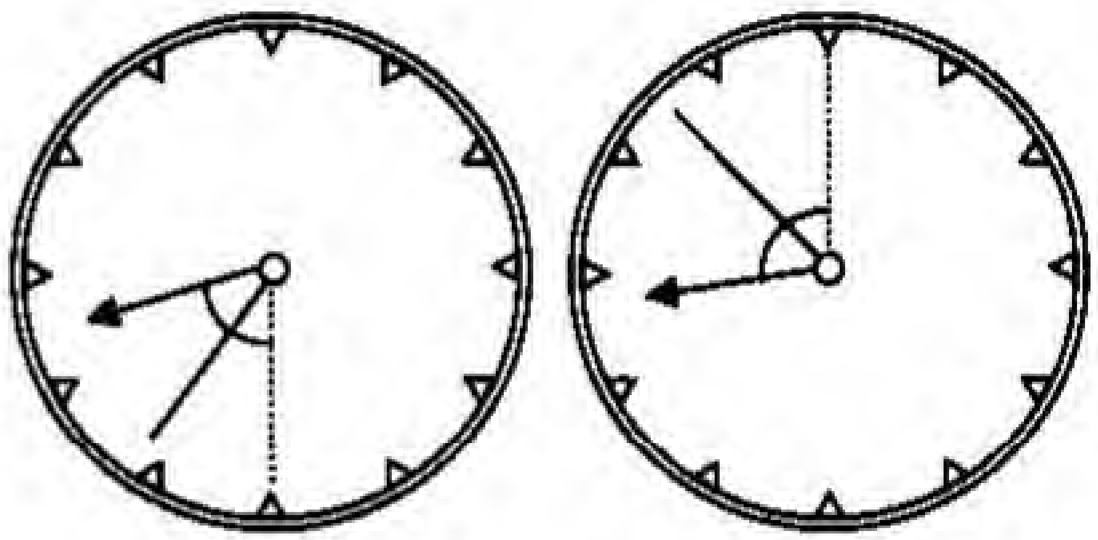
$$\therefore 12(2y-100)=y$$

$$\therefore y=\frac{1200}{23}$$

$\therefore$ The time interval between the two bisection moments

$$= y-x \text{ minutes} = \frac{1200-840}{23} = \frac{360}{23}$$

$$= 15\frac{15}{23} \text{ minutes}$$



87. CALENDAR PUZZLE

Let the top left corner of the rectangle contain date x and the date in the top right corner be $x+a$. Then the dates in the bottom left and right corners will be of the form $x+7m$ and $x+a+7m$. Then we have, $x(x+a+7m)-(x+a)(x+7m)=-7am$ which is divisible by 7.

88. SQUARE YEAR

1936= 44^2. So $45^2 = 2025$, this will be the year wanted in this century. Note that we get this by adding the odd number 89 to 44^2. If you add the next odd number 91 to 45^2, you will get 2116, the only square (46^2) in the 22nd century. Next, to get the only square of 23rd century, we must add 93 to 2116 getting 2209 ($= 47^2$). Continuing in this way, we will get $50^2 = 2500$, while $51^2 = 2601$, showing clearly that the 26th century will have no perfect square number for the first time, i.e. between 2501 and 2600.

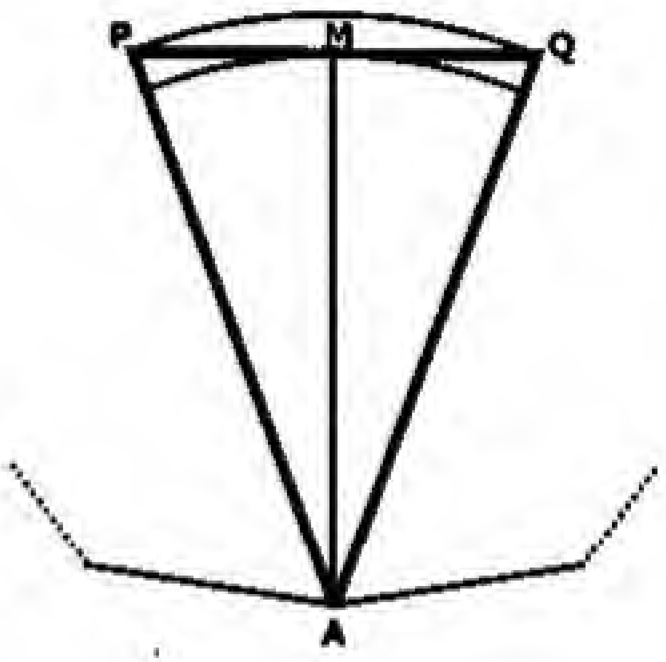
89. HOW MUCH AREA?

Let A be the vertex of the regular polygon, about which the rotation takes place, and PQ the remotest side from A. Draw AM perpendicular to $\overline{PQ}$. P (and Q) will describe a circle of centre A and $\overline{PQ}$ will touch (at M) a concentric circle of radius AM. So, $\overline{PQ}$ will sweep out an annular region between these two circles.

The area of this region is, therefore, $\pi AP^2 - \pi AM^2$

$= \pi PM^2$ (by Pythagoras' Theorem) $= \pi\left(\frac{1}{2}\right)^2$, as PQ = 1 unit.

$\therefore$ The area required $= \frac{1}{4}\pi$.



90. EQUAL WEIGHTS

To keep four objects in each pan, we must keep one object aside. - As the total weight of the nine objects is 45 kgs, what we keep aside must be an odd number of kilograms. So, we have to consider five different cases, as there are five odd numbers, 1, 3, 5, 7, 9. The easiest way of solving the problem is, perhaps, by actual enumeration.

Reject	In one pan	Reject	In one pan
	2,4,9,7		2,9,5,3
1	2,6,9,5	7	4,9,5,1
	2,8,9,3		6,9,3,1
	2,8,7,5		
	4,9,7,1		2,4,7,5
3	6,9,5,1	9	2,6,7,3
	8,7,5,1		2,8,7,1
	2,6,9,3		2,8,5,3
5	2,8,9,1		
	2,8,7,3		
	2,4,6,8		

To explain the above table, let us take one example, say, we reject 5. The sum of the remaining eight weights will be 40 kgs. So, 20 kgs must go in each pan. If 20 is the sum of four weights, two of them must be even or all of them even, so that in the other pan, two will be odd or all odd. So, it is only necessary to show what we put in one pan. In this case, it is easily seen that there are four ways, as shown.

Thus, the total number of ways is seen to be 18.

91. IS IT A SQUARE?

The answer is 'No'. Square every digit from 1 to 9 and take their digital roots. You will always get one of the numbers 1, 4, 9, 7.

Use the result : The digital root of the product of any two numbers is the digital root of the product of the digital roots of those two numbers. As the square of a number is the product of the number and itself, the digital root of a perfect square must be one of the numbers 1, 4, 9, 7. But, the sum of the digits of the given number is 74 and so its digital root is 2, which is not in the list. So the given number cannot be a square.

92. HANGING ROD

The weight of any portion of a rod is proportional to its length.

Let us take the length of the whole rod AB as a unit. Let the length of the portion AC be x, so that BC = $1 - x$. Their weights can be taken as λx and $\lambda(1-x)$. For equilibrium, we must have

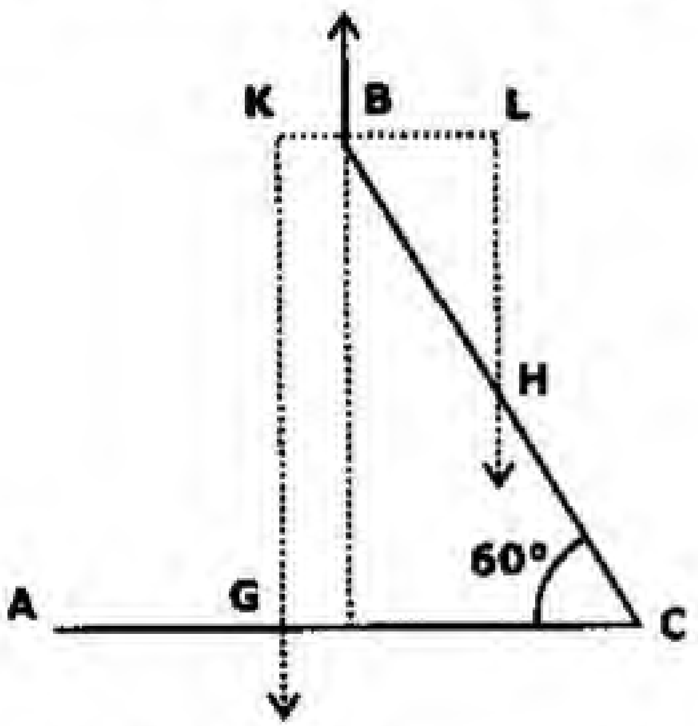
$$\lambda x \,.\, KB = \lambda(1-x)\, BL$$

$$\therefore x\,(GC - BC\cos 60^\circ) = (1-x)\,\frac{1-x}{2}\cos 60^\circ$$

$$\therefore x\left[\frac{1}{2}x - (1-x)\frac{1}{2}\right] = \frac{1}{4}(1-x)^2$$

This gives $3x^2 = 1$ or $x = \frac{1}{\sqrt{3}}$

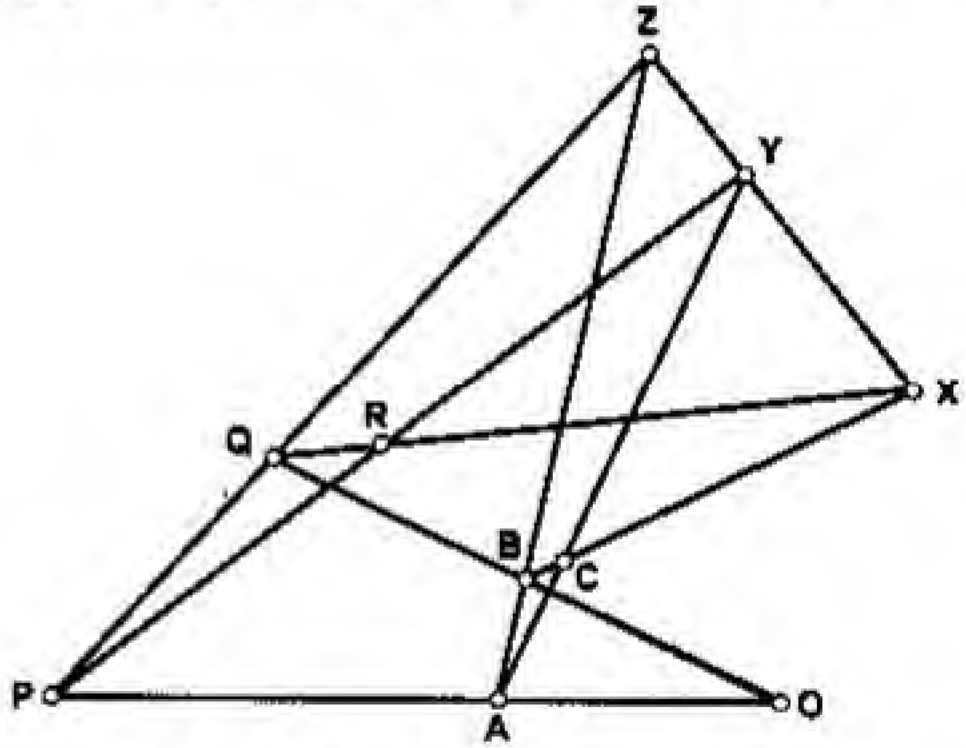
$\therefore$ The point C is such that $AC = \frac{1}{\sqrt{3}}$ AB.



93. (10 - 3) CONFIGURATION

Through any point O draw any three lines OA, OB, OC and on these lines take any three points P, Q, R respectively. Join $\overline{BC}$, $\overline{QR}$ and let them meet (produced, if necessary) in X.

Similarly, $\overline{CA}$, $\overline{RP}$ may be joined and let them meet in Y. Lastly, $\overline{AB}$, $\overline{PQ}$ meet in Z. It will always be found that X, Y, Z lie on one line. This result is known as **Desargue's Theorem.** The figure obtained is Desargue's (10 – 3) configuration. The 10 points are O, A, B, C, P, Q, R, X, Y, Z and there are 10 lines in the figure, namely, OAP, OBQ, OCR, BCX, QRX, CAY, RPY, ABZ, PQZ, XYZ. It is seen that through every point pass three of these lines.



94. DIVISIBILITY TEST

Let N be a given number of many digits. To test for its divisibility by 7, we perform the following operations: Remove the last digit of N and subtract twice the digit

removed from the number that remains, getting a number N_1.

$$\begin{array}{rl} N &= 465814\cancel{3} \\ & \underline{6} \\ N_1 &= 46580\cancel{8} \\ & \underline{16} \\ N_2 &= 4656\cancel{4} \\ & \underline{8} \\ N_3 &= 464\cancel{8} \\ & \underline{16} \\ & 44\cancel{8} \\ & \underline{16} \\ & 28 \end{array} \qquad \begin{array}{rl} N &= 25960\cancel{2} \\ & \underline{4} \\ N_1 &= 2595\cancel{6} \\ & \underline{12} \\ N_2 &= 258\cancel{3} \\ & \underline{6} \\ & 25\cancel{2} \\ & \underline{4} \\ & 21 \end{array}$$

Then, get N_2 from N_1 in the same way as we got N_1 from N.

Then, get N_3 from N_2. and so on. Continue this process until you arrive at a small number which can be easily tested for divisibility by 7.

We have given above two examples in which the numbers 4658143 and 259602 are both divisible by 7, for the final results 28 and 21 are divisible by 7.

Apply the test in a few more examples of your own. If the final result is not divisible by 7, you will find that the original number N also is not divisible by 7.

We now proceed to prove that this will always work. It is enough to show that if N_1 is divisible by 7, then N will be divisible by 7. For, a repetition of the same argument will lead to the desired conclusion. Here is the proof : Let *b* be the last digit of N and after deleting *b*, let A be the number that remains. Then, we may write $N = 10A + b$. Therefore, we have $N_1 = A - 2b$ From this, we get $2N + N_1 = 21A$.

$\therefore$ $2N + N_1$ is divisible by 21, i.e. also by 7.

So, if N_1 is divisible by 7, N must also be divisible by 7.

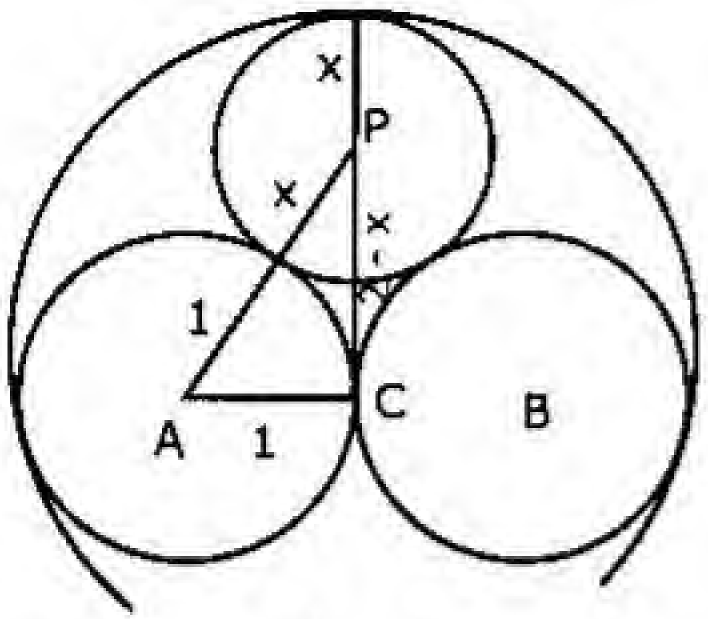
95. CIRCLES IN CONTACT

Let *x* be the radius of the fourth circle (see figure on next page).

Then, using Pythagoras' theorem for the right angled triangle,

we have $\quad (1 + x)^2 = 1^2 + (2 - x)^2$

which gives $\quad x = \dfrac{2}{3}$.



96. PERPETUAL CALENDAR

Given the date, month and year, we have to find the day of the week. Let us first define what we call the month-index and the year-index. The month index is given by the following table.

Index	0	1	2	3	4	5	6
Month	Jan Oct	May	Aug Feb	Feb Mar Nov	Jun	Sep Dec	Apr July Jan

Example - Month index of Aug. is 2, of Sep. is 5.

Year-Index is calculated as follows : Write down the following four numbers, (assuming the year number has four digits). (a) The number formed by the last two digits (b) The number

formed by the remaining digits (call it century number) (c) Quotient of year number ÷ 4(d) Quotient of century number ÷ 4. Adding these (a, b, c, d) you will get the year index. Example :

Year 1837	18	Century number
	37	Last two digits
	459	Quotient of 1837 ÷ 4
	4	Quotient of 18 ÷ 4
Total	518	Year index.

Formula: Given the date, month and year, add the date, month index and year index. Then divide the sum by 7. If the remainder is 1, it is Sunday, if 2 Monday, ..., if 6 Friday and if 0 Saturday.

Example : What day was 15th June 1933 ?

It was a Thursday, as the calculation shows.

15	Date
4	Month index of June
19	
33	
483	→For year -index of 1933
4	
7 | 558	
79	
	- remainder 5 (Thursday)

A second example : What day is 1st Dec 2345 ?

	1	Date
	5	Month index
	23	For year-index
	45	
	586	
	5	
7	665	
	95	- remainder 0 (Saturday)

What day were you born ? Verify with your birth-date !

(Important note : In the month index table, the italicised months *Jan* and *Feb* are to be used only in the case of a leap year. No change for the other months of a leap year)

97. CUT NUMBER

Could you find the next cut number after 6 ? It is 35. We have, in fact,

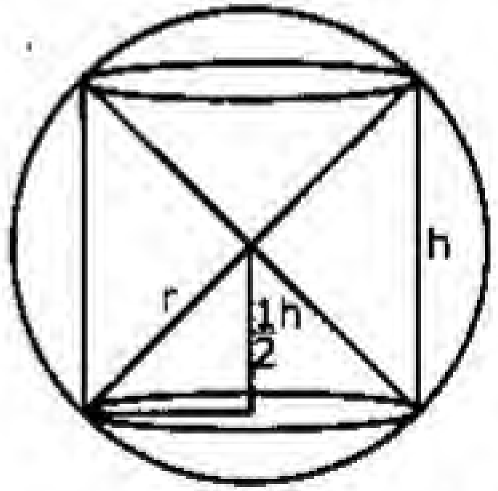
$$\underbrace{1 + 2 + \dots + 34}_{595},\ 35,\ \underbrace{36 + 37 + \dots + 49}_{595} \qquad 595 = 595$$

for $\frac{34 \times 35}{2} = \frac{14}{2}(36 + 49) = 595$

98. A CURIOUS RESULT IN AREAS

The areas of the semicircles are proportional to the areas of the squares drawn on the sides of the triangle. Therefore, by Pythagoras' theorem, the area of the largest semicircle is equal to the sum of the areas of the other two semicircles. Removing the common region, we get the result stated. Note that the region with the horizontal stripes occurs twice in the reckoning and so it has to be subtracted.

99. A HOLE THROUGH A SPHERE



Let the radius of the ball be r and the height of the cylindrical hole be h.

$\therefore$ The radius of the cylinder is $\sqrt{r^2 - \frac{h^2}{4}}$

The height H of the segment $= r - \frac{h}{2}$

Volume of the sphere $= \frac{4}{3}\pi r^3$

Volume of cylinder $= \pi\left(r^2 - \frac{h^2}{4}\right)h$

Volume of the two segments at the two ends

$$\frac{2}{3}\pi H^2 (3r - H) = \frac{1}{12}\pi (2r - h)^2 (4r + h)$$

$\therefore$ After scooping out, the volume of the remaining portion of the ball

$$= \frac{4}{3}\pi r^3 - \pi\left(r^2 - \frac{h^2}{4}\right)h - \frac{\pi}{12}(16r^3 - 12r^2h + h^3)$$

$= \frac{1}{6}\pi h^3$ which is independent of r.

$= 288\pi$ c.c., if h = 12 cms.

100. AVERAGES

Let there be n numbers $x_1, x_2, \ldots, x_n$. Their average is $\frac{1}{n}(x_1 + x_2 + \ldots + x_n)$. Considering all the pairs, their number will be $\frac{n(n-1)}{2}$. The mean of every pair is half of their sum. Before taking the mean, let us write down the $\frac{1}{2}n(n-1)$ terms as they are. In this each particular number x_i will, obviously occur $(n-1)$ times. Their sum will be $(n-1)x_i$. There will be a denominator of 2 while taking the mean and a denominator of $\frac{1}{2}n(n-1)$ while taking the average of the means.

Ultimately, each number x_i will appear as

$$\frac{(n-1)\,x_i}{2} \div \frac{1}{2}n(n-1) = \frac{x_i}{n}$$

$\therefore$ the average of the means $= \dfrac{1}{n}(x_1 + x_2 + \ldots + x_n)$

= the original average of the numbers.

101. MEANS AND MISSING NUMBERS

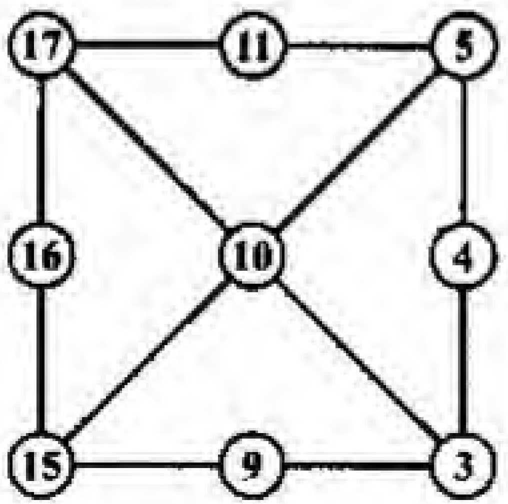
Let us take x as the number in the cell at the centre. Then the other entries are as follows:

Bottom right : $2x - 17$
Bottom left : $2 \times 9 - (2x - 17) = 35 - 2x$
Top right : $4 \times 2 - (2x - 17) = 25 - 2x$

$\therefore 2x = (35 - 2x) + (25 - 2x)$

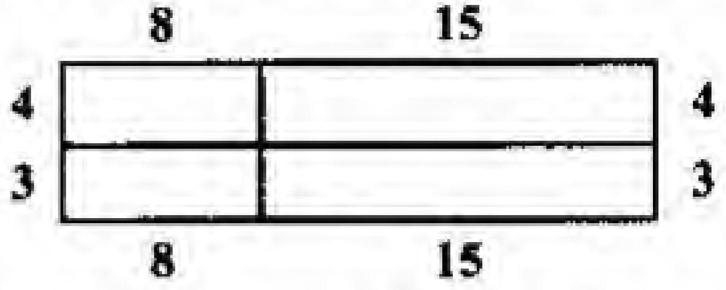
$\therefore x = 10$

Substituting →



102. AREA OF THE FIELD

The common factor as mentioned in the Hints, is 4. So the linear dimensions can be taken as shown in the figure below:



The required area is therefore 45 sq. mts.

103. WEEK DAY REPEAT

Very easy. Solution is almost there in the Hints. However, refer to problem 96. The difference comes from the two year-indices only.

18	22
82	82
4	5
470	570
574	679

The difference is always 105, which is a multiple of 7.

104. TAMPERING WITH TIME

Raise the book and hold this page in a vertical plane before your eyes. Turn the page - back to front (to secure mirror effect) and see the trace of the figure against a bright background. Then turn the page in its plane through 45° in the anticlockwise sense. You can, now, actually read the correct time! It is 2:35.

105. SQUARE WITHIN A SQUARE

Area of the bigger square is $(a+b)^2$ and that of the smaller one is $c^2 = a^2+b^2$. So we have $(a+b)^2 = 1.6\,(a^2+b^2)$.

This gives $3a^2-10ab + 3b^2 = 0$ or $3\left(\frac{a}{b}\right)^2 - 10\left(\frac{a}{b}\right) + 3 = 0$.

So that $\frac{a}{b} = 3$ or $\frac{1}{3}$. So the smaller part is $\frac{1}{3}$ the bigger part. You may check this by taking $a=1$, $b=3$.

106. COMPARISION OF AREAS

As suggested in the Hint, note first that the area of each of the sixteen squares is $\pi\left(\frac{a}{8}\right)^2$ and that of each of the nine squares is $\pi\left(\frac{a}{6}\right)^2$.

So the area of the shaded region in the first square =

$$a^2 - 16\,\pi\left(\frac{a^2}{64}\right) = a^2\left(1 - \frac{\pi}{4}\right)$$

And that in the second square is $a^2 - 9\,\pi\left(\frac{a^2}{36}\right) = a^2\left(1 - \frac{\pi}{4}\right)$.

So the two areas are equal.

107. A CHARACTERISTIC OF 4 x 4 MAGIC SQUARES

x	x	x	x
	xx	xx	
	xx	xx	
x	x	x	x

	xx	xx	
	xx	xx	

Denote the constant sum by S. The sum of all the entries descried in the Hints will be 4S. Then remove the entries in the top and bottom rows whose sum is 2S. What remains are two equal entries in each of the central squares, and their sum is 2S. Thus, the sum of the single entries in the central squares will be S.

108. SERIES SUMMATION

The sequence of sums is as follows

1, -1, 2, -2, 3, -3, 4, -4 ....

Here, obviously, every natural number will occur after twice the previous lower numbers have been written down. So, -2006 is 4012^{th} number. Therefore 2007 is 4013^{th} number. Hence, the number of terms required is 4013.

109. OVERTAKING PROBLEM

If the cyclist is C, and the motorist M, then they are both driving their vehicles on $\overline{AB}$ as shown in the figure above.

C took 330 minutes to cover the distance AB (6:20 a.m. to 11:50 a.m.).

M took 110 minutes to cover the same distance (7:30 a.m. to 9:20 a.m.).

∴ M's speed is 3 times C's speed.

Let P be the place where C was at 7:30 a.m.and X the place where M overtook C.

To cover the distance AP, C took 70 minutes (6:20 a.m. to 7:30 a.m.). ....(i)

Time taken by M to cover the distance AX

= time taken by C to cover the distance PX

∴ AX = 3PX

∴ AP = 2PX

∴ to cover the distance PX, C took 35 minutes (from (i)).

∴ to cover the distance AX, C took 70 + 35 = 105 minutes.

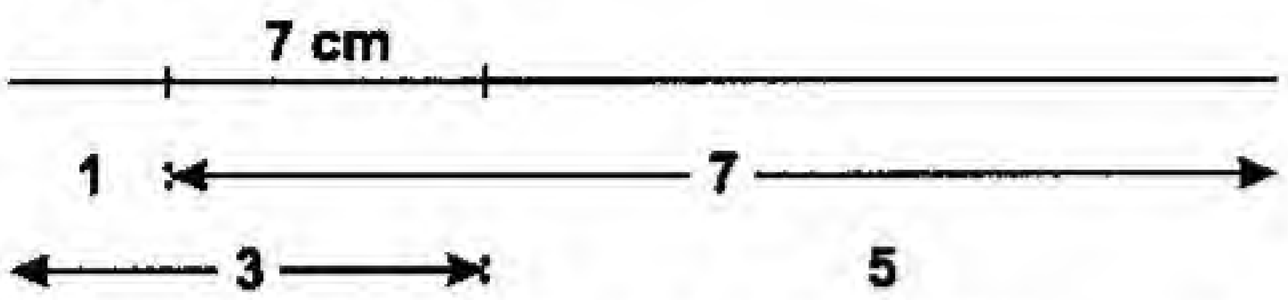
∴ C was at X at 8:05 a.m. (105 minutes after 6:20 a.m.)

∴ answer – 8:05 a.m.

110. ADULTERATION

5.4 % of 25 is 1.35. This is the volume equivalent of fat. This will be 5 % of what ? The answer is simple. It is 27. So the new volume of adulterated milk is 27 litres. So the amount of water added to milk was 2 litres.

111.ROD ON A SEGMENT

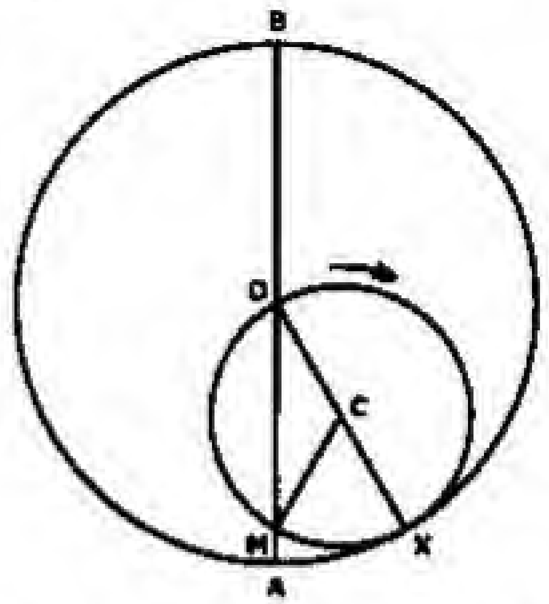


We are given that the first ratio is 1:7. This means that if the length of the line segment is 8 units, the left hand portion is 1 unit. Similarly, the second ratio is 3:5. It means that if the length of the line segment is 8 units, the extreme right hand part is 5 units. Then out of 8 units of total length, the rod is $8 - 1 - 5 = 2$ units long. But it is given to be 7 cms. So the whole length of 8 units is 28 cms.

112. SUMS OF SQUARES

Take any composite odd number. For instance,
$15 = 5 \times 3 = 15 \times 1$. Hence, $(4+1)(4-1) = (8+7)(8-7)$.
So that $4^2 - 1^2 = 8^2 - 7^2$.
By transposition, $4^2 + 7^2 = 8^2 + 1^2 = 65$.
Similarly, $21 = 7 \times 3 = 21 \times 1$.

113. ROLLING DISC

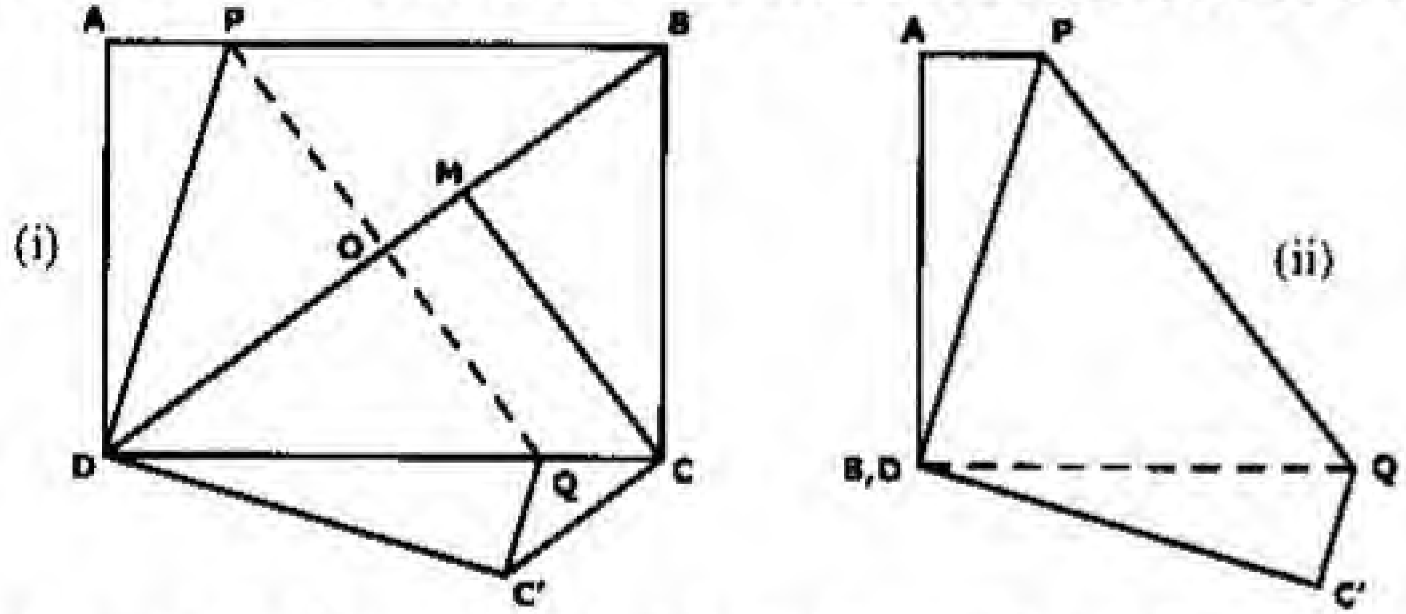


Let the marked point M on the edge of the disc be initially at A on the circle. Now, the disc has rolled to a new position (in a clockwise direction) with X as the point of contact and M has moved up. The successive points of contact between the disc and the circle being the same, clearly, the arcs AX and MX are equal in length. So, as the radius of the circle is twice that of the disc, we have $\angle MCX = 2\angle AOX$. This is possible only if the variable point M is on the fixed line AO. Thus, as the disc rolls to make two full revolutions, the path of M will be diameters $\overline{AB}$ and then $\overline{BA}$.

114. MULTIPLES OF 11

The answer is none. Proof: The sum of all the six digits is 21, which is odd and so one condition of equality referred to in the test cannot hold. In the other possibility, obviously, the two sums will have to be 5 and 16. But evidently, no three digits will add up to 5 as the least such sum is $1+2+3 = 6$. So there can be no multiple of 11, in this question.

115. PROBLEM OF FOLDING A RECTANGULAR PAPER



The crease is shown by PQ (dotted) meeting BD at O and C' is the new position of C. BO = OD. Let CM be perpendicular to BD. Now, BD=25 units (by Pythagoras' Theorem). Also, $BC^2 = BM.BD$

$\therefore 15^2 = BM.25$

$\therefore BM = 9.$

$\therefore OM = 3.5.$

$\therefore CC' = 2(0M) = 7$ units.

116. NUMBER OF PRIZES

Suppose they get A, B, C and D number of prizes where $A<B<C<D$. If A=1, D=7. So we have $1<B<C<7$.
Now B or C cannot be 4, otherwise, $4-1 = 7-4 = 3$. So B or C can be 2, 3, 5 or 6.
If B = 2, $C \neq 3$ and $C \neq 6$, otherwise difference $3-2 = 7-6 = 2-1 = 1$.
So C = 5, giving (A,B,C,D) = (1,2,5,7).
We may check the differences.
Similarly, B=3 implies $C \neq 2$, $C \neq 5$.
So C = 6, giving (A,B,C,D) = (1,3,6,7). But in the first case $1+2+5+7 = 15$, in the second case, $1+3+6+7 = 17$. To get 33, we need to add 18 in the first and 16 in the second, distributed equally among A,B,C and D. This is possible only in the second case, giving the four numbers as 5,7,10,11.
So the four girls got 5,7,10 and 11 prizes.

117. ARITHMETICAL CONUNDRUM

We have to solve the alphamatics

TWO x TWO = OOOOO.

Now, OOOOO = O x 11111

= O x 41 x 271

Since the left side shows multiplication of two three-digit numbers, the letter O represents the digit multiplying 41 (and not 271). If we take 271 as TWO, then T = 2, W = 7.

∴ WIT = 712 and there is no one digit multiplying 41 which will give 712. Thus WIT = 271 giving W=2, T=1. So, O x 41 = 120, from which it is clear that O=3.

∴ we have 123 x 271 = 33333.

To answer the question, what is IT, we get

IT = 71.

COMMENTS

1. MILKMAN'S PROBLEM

It should be obvious that the milk can be divided between two customers not only into (4, 4), but also, if desired, into (1,7), (2, 6) or (3, 5). Can you divide 24 litres of milk into (12, 12) litres by using two empty vessels of capacity 17 and 7 or 13 and 11 litres ? What is the general method of solution ?

Also give a graphical solution of this problem on the same lines as that given in the solution.

2. UNICURSAL FIGURE

This is a very simple example of unicursal networks. It is always possible to draw a diagram in a single circuit, provided that the number of lines emanating from each point is even or there are at most two points for which this number is odd.

3. NECKLACES

All necklaces except two are symmetrical. Which two are these ? Solve the problem of seven bead necklaces, when the beads are of three colours black, red and white.

4. MAGIC SQUARES

The same thing can be done for magic squares of higher orders. However, if the entries are to be natural numbers, there is a restriction on the constant sum to be prescribed. What is this restriction ? Construct a four-by-four magic square with constant sum 130.

6. ZEROLESS FACTORS

Any arbitrary power of 10 will not have this property. The 9th power happens to have. There is another bigger power of 10 with this property. Can you find it ?

7. FIND THE NUMBER

Extend the problem by stipulating instead of "divisible by 11" add "divisible by 13 and leaves remainder 9 when divided by 11".

8. AGE PROBLEM

Notice that *x*, my son's age, may be any number you like. It is absolutely arbitrary. "12 years" will always be the answer. Let us verify by taking my son's present age as say, 13. 4 years hence, his age will be 17. 4 years ago my age was 34 (= 17 x 2). So, my present age is 38. 12 years hence, our ages will be 25 and 50 and we know that 25 x 2 = 50.

9. WATCH PROBLEM

There are several variations of this problem. One can find the exact time between 3 o'clock and 4 o'clock when the two hands are in a line or coincident. Also, the stipulation 'after 3 o'clock', can be changed into 'after 4 o'clock' or any other time.

10. EQUALISE AMOUNTS

It will be interesting to verify the answer. Here are the steps.

A	B	C	
29	24	22	
– 13	+ 13	–	
16	37	22	After A gives B
–	– 12	+ 12	
16	25	34	After B gives C
+ 9	–	– 9	
25	25	25	After C gives A

Actually if the word 'least' is removed from the problem, we need not take $t = 1$ and the problem has an infinite number of solutions. For example, if $t = 3$ we have $a = 91$, $b = 76$, $c = 70$. Now, verify that this is correct as above.

There is another method of solution. Assume the final amounts as x, x, x. Then work backwards. You will have the initial unequal amounts as expressions in x which appear like fractions which should really be integral. Thus, the appropriate value of x can be found and the original unequal amounts derived. This is equally interesting. Try it.

There are variations and generalizations of this problem.

(i) If there are four persons A,B,C and D and the transactions are as before, solve the problem. A student of mathematics can try for the general case, for n persons.

(ii) Instead of taking 'one person gives another person half as much as the second one and a rupee more', take 'one person gives another person one-third as much as the second one and two rupees more'.

(iii) Instead of "A gives B, B gives C and C gives A", take "A gives B and C, B gives A and C and C gives A and B". It can be seen that many such variations can be had.

11. LADDER PROBLEM

We have rejected the other root (2.5 meters) of the equation because it is less than 6. But CA = 2.5 meters also gives a possible position of equilibrium for the ladder. This other position can be seen by turning the figure through a right angle, making CB the ground and CA the wall. Thus, CB = 2.5 meters.

12. WRONG MAN OUT

There is another way of seeing how figure 6 is different from all the rest. Check congruency by superposition. You can imagine each figure moved in the same plane and made to coincide with any other figure except figure 6 which must be taken out of the plane and turned over before superposition.

13. MISSING NUMBERS

How does one find from the given data the formula (here *ab-a-b)* which will satisfy all the given entries? There is no method. But, could you strike at the formula?

This is just a problem to test one's keen sense of observation and number relations.

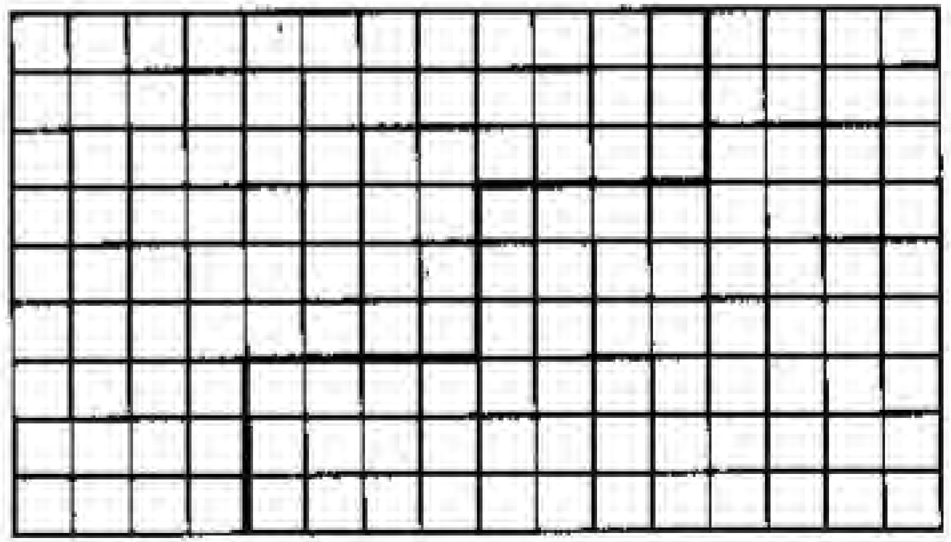
15. TWO BOATS

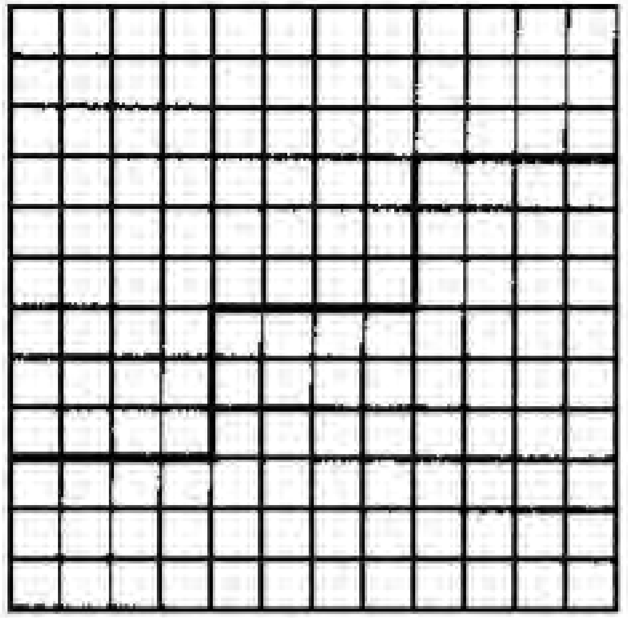
If the boats continue to shuttle indefinitely between the banks, their third meeting will take place at the other trisection point between the banks. Indeed, every successive meeting will take place at one of these three points only, the two trisection points and the middle point.

16. FOUR CHILDREN

It is easy to do this by 'Permutations and Combinations'. Because in disposal (a) there are ${}_4C_2$ ways and in (b) ${}_4C_2 \times 2$ ways that is 6 and 12 ways respectively.

18. DISSECTION PROBLEM





You can form a square with two pieces cut out of a rectangle whose length and width are not only 9 and 4 but also from another rectangle of sides 16 and 9 (as shown in the figure above) indeed in general, the rectangle can have its length and width $(n + 1)^2$ and n^2 units, where n is any natural number.

Problem number 63 is general and more mathematical.

19. BLACK AND WHITE CAPS

There are other variations of this problem. Consider this: There were three ladies only in a compartment of a train which was running into a long tunnel. The engine driver inadvertently, let the smoke out, which naturally entered the compartments profusely. As soon as the train emerged out of

the tunnel into broad day-light, each lady saw the faces of the other two blackened by soot. All the three started laughing simultaneously at the other two. Prove that the cleverest of them is the one who first stops laughing.

This is due to the sudden realisation of the fact of one's own face being black. How does this realisation come ? Let C be the lady who first stops laughing. She would argue as follows :

Suppose my face is clean. Then, one of the other two, say, B could have had a similar realisation and could have argued within herself "C's face is clean. So my face must be black. If not, what was the third lady A laughing at?"

Unlike the prisoners' problem, in so far as the premises are concerned, the three ladies are on par and any one of them could have argued as above. But, the reasoning would occur to the cleverest first and so she would be the first to stop laughing. Moreover, there is nothing here corresponding to five caps. Only, the essence of the logic is the same.

A generalisation to more than three ladies is possible. Think of the problem of four ladies.

20. WHAT IS PLUNKY?

We hope there is no controversy in this problem. It is a matter of close observation.

21. SPECIAL TRIPLETS

There is a stronger result than what we have proved. If six points are joined by all the possible 15 lines coloured black and red arbitrarily, there will always be not one, but two unicolour triangles (both red, both black or one red and one black). But, the proof is not quite easy. You may congratulate yourself if you find a proof.

23. DIVISIBLE BY 7

The number is clearly divisible by AB (which is really 10A + B). Can you visualise the actual division of the number by AB yielding the quotient 10101 which is 7 x 1443?

The number has also other factors namely, 3, 13 and 31.

24. MAKE A SQUARE

It is not impossible to get the square by trial and error, if one has the patience. But, the geometrical reasoning given in the solution is instructive. One perhaps easily gets a rectangle

first and mistakes it for a square as the length is only slightly longer than the width.

25. BIRD AND SNAKE

In Lilavati : Bijganit, there is a problem very much resembling this one.

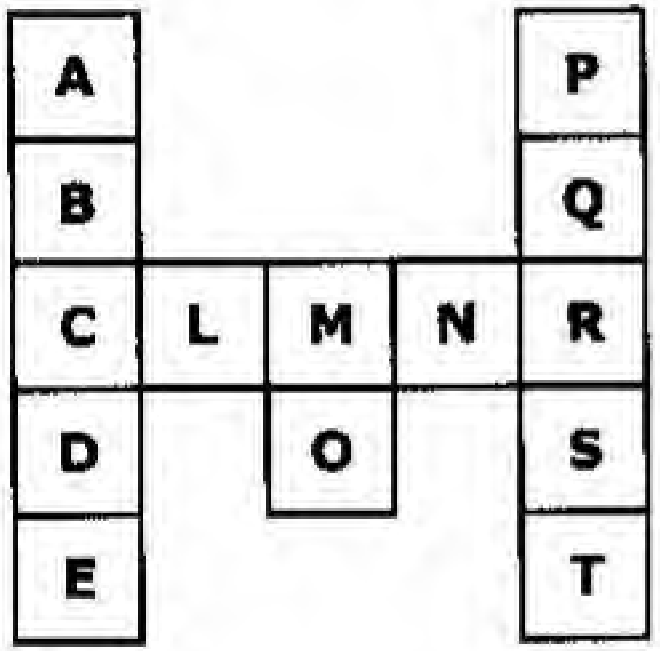
26. 12 BALLS PROBLEM

The problem has the merit of admitting a clear logical analysis. It can also be generalised. But, we can't give the generalisation. However, readers may be interested in this question : If four weighings are allowed, from among how many maximum number of equal balls can one faulty ball be picked out? What if five, six etc. weighings are allowed?

The 12 balls problem is known to admit computerised treatment. We have devised three new interesting gadgets including one based on the principle of window reader and another on punched cards which give the solution without the aid of human thinking. It is surprising that the intricate logical steps of the solution involving the mental process can be mechanised.

27. PARKING PUZZLE

Were there 17 moves in your solution, as in the given solution, or more? If you can exercise a little more patience, you may try to solve the same problem with 5 coins (instead of 3) on each side and the parking places are as before. We are giving the solution below. But don't look at it before you make an independent attempt. The problem is solvable in 89 moves even if there are seven coins on each side.



RO	SL	DO	QT	CR
CR	OS	EM	MQ	DN
DN	QO	BE	LS	OD
OD	PM	MB	CR	NC
BO	SP	ND	BN	RL
AM	MQ	RC	OB	SO
DA	LS	QL	NC	LS
MB	CR	OQ	RL	CR
ND	BN	SO	QO	OC
RC	OB	TM	LQ	

28. MAGIC THIRTEEN

It appears that there are just three other solutions besides the one already given. They are shown here. The central number is always 7 as already shown. You may write the remaining twelve numbers in any one of the following orders on the periphery,

(i) 1, 8, 3, 10, 9, 2, 13, 6, 11, 4, 5, 12

(ii) 1, 12, 8, 10, 3, 5, 13, 2, 6, 4, 11, 9

(iii) 1, 8, 12, 4, 5, 3, 13, 6, 2, 10, 9, 11

29. EXITS & ENTRANCES

Compare this problem with the unicursal figure (no.2). Represent each room by a point and the entire outside also by a point. Designate a room and the corresponding point by the same letters. Thus we have seven points A, B, C, D, E, F, G. Represent a doorway by a line joining the two points representing the two rooms connected by the doorway. Our problem then is equivalent to drawing the network unicursally that is in a single circuit, without retracing any line. The principle mentioned in the comments on problem - 2 is applicable here. Thus, you must start from a point from which an odd number of lines emanate and end at a similar point.

Correspondingly, your path must start from inside a room which has an odd number of doorways and end in a similar room.

30. TROMINO PUZZLE

First note that any unit square from a 2 x 2 square can be deleted to leave a tromino which we shall call a unit tromino. Next take a 4 x 4 square and delete any unit square. The square so deleted will lie in one of four 2x2 corner squares, which will accommodate a unit tromino as above. The remaining portion is a big tromino which can accommodate 4 unit trominoes.

Then take a 8 x 8 square and delete any unit square. The square so deleted will be in one of the four 4x4 corner squares. First fill up this 4x4 square minus the unit square with five unit trominoes as above. The remaining portion can accommodate 16 unit trominoes. For doing this, place the first unit tromino which is made up of three unit squares one from a corner of each of the remaining 4x4 squares and then as before-5 unit trominoes in the remaining portions of each of the 4x4 squares.

This process can be obviously extended to bigger and bigger squares. The next case will be that of a 16 x 16 square with any one unit square deleted and 85 unit trominoes accommodated.

33. DEVOUT PRIEST

This problem is a slight variation of another well known problem similar to this in which the magical property of the pond is that it doubles the number of flowers in the basket. The solution in this case is $x = 8$ and $y = 7$. The origin of the problem is not known. This problem or our problem can be extended.

Take our problem and suppose there are four ponds. Now, we should not equate $\frac{27x}{8} - \frac{19y}{4}$ to zero, but go ahead one step further and equate $\frac{3}{2}\left(\frac{27x}{8} - \frac{19y}{4}\right) - y$ to zero.

This will give $x = 130$ and $y = 81$.

Let us verify :

$$130 \to 195 - 81 = 114 \to 171 - 81$$
$$= 90 \to 135 - 81 = 54 \to 81 - 81 = 0$$

It is not difficult now to extend the problem to five temples and five ponds. A mathematician may like to generalise the problem as follows: Let us first take the case of three temples and three ponds. The problem is same as before except that when the basket of flowers is dipped in a pond, the number of flowers become p fold. Working as before we have $p[p(px-y)-y]-y=0$ which gives $p^3x=(p^2+p+1)y$. We may take as the values of x and y, p^2+p+1 and p^3 respectively provided that these are integral. Otherwise, you must multiply them by d^3 where d is the denominator of p. Thus the least integral solution is $x=d^3(p^2+p+1)$ and $y=d^3p^3$. In the original well known problem $p=2$, $d=1$, giving $x=7$ and $y=8$. In our

problem, $p=\frac{3}{2}$, $d=2$ giving $x=38$ and $y=27$.

We may generalise this still further and stipulate n temples and n ponds. In this case, the solution is easily seen to be

$$x=d^n(p^{n-1}+p^{n-2}+\ldots+p+1)=d^n\frac{(p^n-1)}{pn-1} \text{ and } y=d^np^n.$$

The problem can be made more versatile by saying that the ponds have different magical properties and the offerings in the temples are also different. Let us illustrate this by an example.

Suppose we have three temples and three ponds. The first pond increases the flowers by 100%, the second by 50% and the third by 25%. The priest offers in the first temple some flowers, in the second twice the number and in the third four times whatever was offered in the first. The question is how many flowers did the priest start with and how many did he offer in the first temple ? The general formula we got cannot be applied here, but the method of work is similar. The answer to this problem is $x=67$; $y=30$. Verify: $67 \rightarrow 134-30 = 104 \rightarrow 156-60=96 \rightarrow 120-120=0$. There is no end to a wide variety of similar problems you can create. Create one of your own and try it on your friends.

34. EAST - SOUTH ROUTES

In the Hints, we said that a student of mathematics can use "permutations" to solve this problem. Note, first that if we call a move from any point to an adjacent point (east or south) a "step", then in every route there are just six steps of which three are eastwards and three southwards. Our problem is then equivalent to this. In how many ways can six letters, of which three are E's and the other three are S's be permuted ? We know that the number of permutations of n things, p of which are alike of one kind, q of which are alike of a second kind and so on is $\frac{n!}{p!\,q!}$. So, in our case the answer is $\frac{6!}{3!\,3!}=20$.

Beginners may like to see actually these 20 arrangements. Here they are :

EEESSS	SEEESS	SSEEES	SSSEEE
EESESS	SSEESE	SEESES	EESSES
SEESSE	EESSSE	ESEESS	SSESEE
SESEES	ESSEES	SESSEE	ESSSEE
ESESES	SESESE	ESSESE	ESESSE

The specimen routes given in the problem correspond to three of these arrangements, namely ESESSE, SSEEES and SEESSE respectively.

We may extend this problem as follows :

Given 20 points in a rectangular formation of 4 rows of 5 points each, in how many different ways can one go from the point A to point B, moving, as before only eastwards and southwards. If you want to do this by actual ennumeration, first note-that in every route, there are seven steps which will correspond to four E's and three S's. The number of different ways will be equal to the number of permutations of seven letters - 4E's and 3S's. This number is $\frac{7!}{4!3!} = 35$. One may write out a complete list of the 35 arrangements.

In a general form of the problem, we have a $m \times n$ rectangular array with n rows of m points each. The question is, in how many different ways can we go from the top left corner point to the bottom right corner point, taking only eastward and southward steps ?

The answer is $\frac{(m+n-2)!}{(m-1)!(n-1)!}$

35. AN UNKNOWN DIGIT

As an example of another problem of this kind we may solve the following : All the digits, except the middle one of a number N of 999 digits are 4's and N is divisible by 13. Find the middle digit.

As before the number 444444 is divisible by 13. Here too, we may delete in N from both ends 498 digits each (i.e. really 2 x 83 blocks of 444444) without affecting the property of divisibility by 13. We will then, be left with a three digit number of the form $4x4$, where x is the unknown middle digit. If this is divisible by 13, we must obviously have $400 + 10x + 4$ or $404 + 10x$ divisible by 13. Subtracting from this 403 which is a multiple of 13, we conclude that $10x + 1$ is a multiple of 13. Hence x can only be 9. Thus, the required middle digit is 9.

36. WRONG LABELS

This is an extremely simple problem with a nice little bit of logical reasoning. We had said in the Hints that there are only two ways of wrongly labelling the boxes. Alternatively, since we can see the labels, there are only two possibilities for the contents as shown below :

Labels (wrong)	WB	WW	BB
True contents	WW	BB	WB
True contents	BB	WB	WW

which perhaps makes the solution more obvious. The solution of an extended problem is not so easy. The problem is : four closed boxes contain three balls each of either colour black or white, all wrongly labelled WWW, WWB, WBB, BBB. How many draws are necessary to enable one to discover the contents completely ? Perhaps, the answer will depend on a knowledge of the colour of the balls previously drawn. In other words, can the balls be drawn one by one or should all the draws be decided in advance ? It appears that the minimum number of draws is four and maximum six. As the wrong labels can be seen, the number of possibilities for the true contents are found to be nine. These are given below schematically for those who want to tackle the problem fully.

	Box 1	Box 2	Box 3	Box 4
Labels (wrong)	**WWW**	**WWB**	**WBB**	**BBB**
True contents	WWB	WWW	BBB	WBB
"	WWB	WBB	BBB	WWW
"	WWB	WBB	WWW	BBB
"	WBB	BBB	WWW	WWB
"	WBB	BBB	WWB	WWW
"	WBB	WWW	BBB	WWB
"	BBB	WBB	WWB	WWW
"	BBB	WBB	WWW	WWB
"	BBB	WWW	WWB	WBB

Another type of extension of the problem is when we have three colours white, black and red. We may have two balls in each of six closed boxes with wrong labels : WW, BB, RR, BR, RW, WB. Formulate a similar problem and solution. The problem becomes extremely complicated with an increase in the number of balls in each box or increase in the number of colours.

37. DOUBLE AMOUNT

You can formulate your own problems of this kind. For example, replace 9 and 5 rupees by say 13 and 7 rupees, or any other pair of numbers.

You may solve this always by a set procedure as follows: Add the two figures 13 and 7. You get 20. Multiply by 3. Get 60. Subtract the difference between 13 and 7, namely 6. Get 54. Half of this, which is 27 is Arun's money in rupees. Half of 60 + 6 = 33 is Ashok's money in rupees. Verify. Do the original problem by a similar procedure. Can you explain why this procedure will always work? Also can you find a procedure if, in a similar problem, after the transactions each of them has not twice but thrice or four times what the other has ?

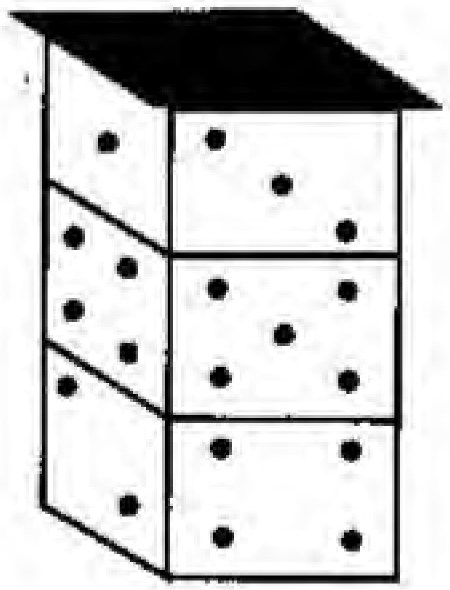
39. FARMER'S WILL

This is a wellknown and a famous problem. The judge's solution is regarded by many as ingenious. We have included this here in order to have an opportunity of recording a protest and of saying that as fractions of a cow are inadmissible, the problem has really no solution. The judge's solution is unmathematical and illogical. Unless the fractions representing the three shares total upto 1, the partitioning is impossible.

Did the farmer not realise that 1/2 + 1/3 + 1/9 is not 1 and did the judge not know that to add and then take back his own cow is illegal ? If adding anything and later retrieving the same thing can be allowed, may I suggest the following ridiculous problem ?

A farmer had 7 cows and two sons. He wrote in his will that 1/4 of the cows should go to his first son and 1/10 to his second son. The judge who was asked to help in the partitioning, brings from somewhere or his own 13 cows and adds them to the farmer's 7 cows and makes a total of 20 cows. He gives 1/4 of 20 or 5 cows to the first son and 1/10 of 20 or 2 cows to the second son. He then takes back his own 13 cows. Will you accept this solution ?

40. A DICE TRICK



research and prove, for instance, that if there are m points in each of n radial lines, a necessary condition for the design to be unicursal is that m and n should be prime to each other. On the other hand, if h is the number of circuits in a design, how is h related to the HCF of m and n?

43. AN AGE PROBLEM

This is a common-sense question. Some solvers might start assuming the age of Maya (or Dora) as x and try to solve an equation. But no algebra is needed. Note it is immaterial when Maya said to Dora "17 years ago, I was as old as you will be 17 years hence". She can truthfully say this at any time if she can say this once. Do you see why?

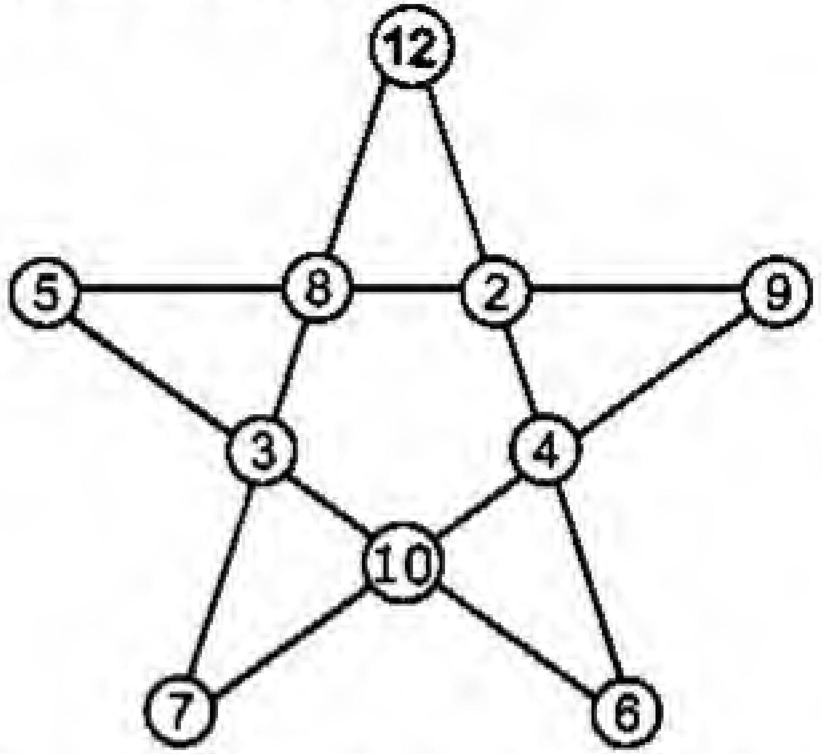
44. REVERSED DIGITS

Try the same problem, with the multiplier 9 in place of 4. That is, solve the alphamatics :

ABCDE x 9 = EDCBA.

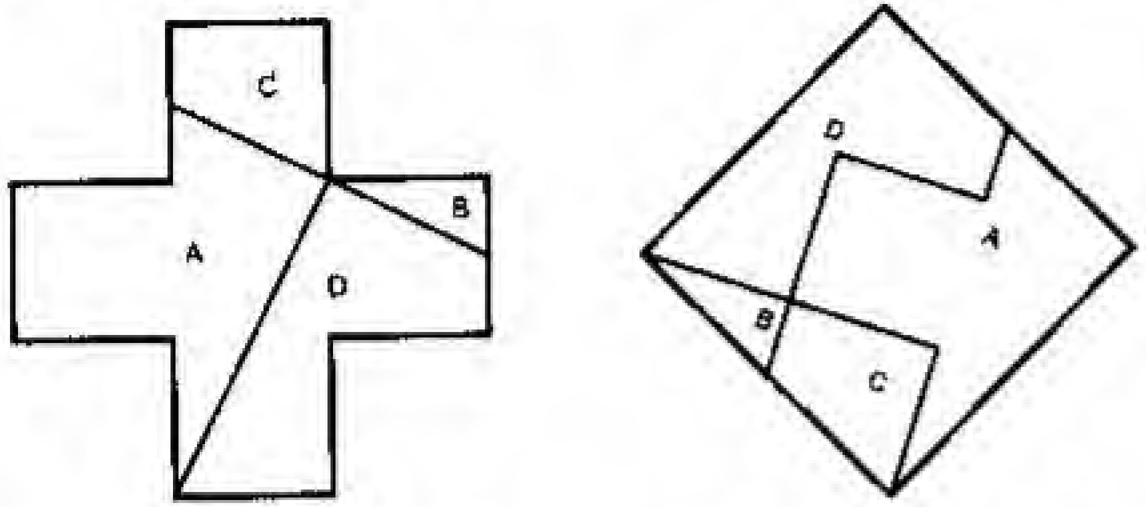
Here all the digits may not be different.

45. PENTAGRAM NUMBERS



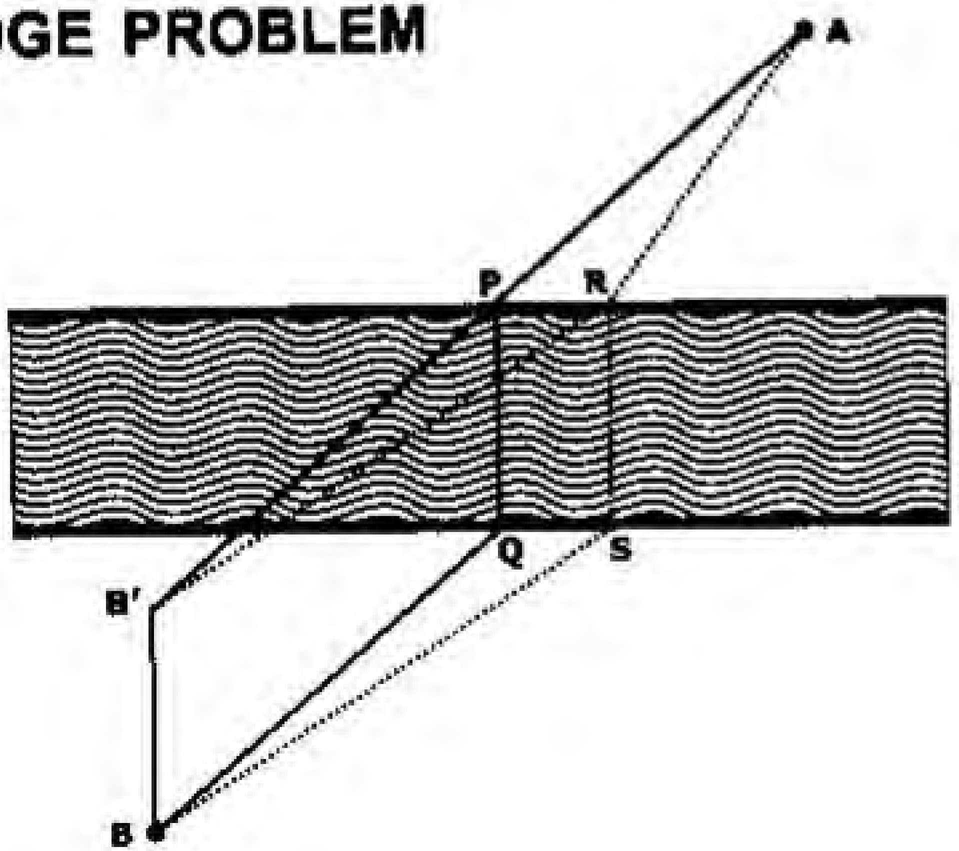
Can you find the reason why in this problem, we gave the numbers 1 to 12, dropping 7 and 11, instead of the first ten natural numbers 1 to 10 ? We can beautifully exhibit the arrangement in the solution by placing the ten numbers at the points of a pentagram. As the sum of the four numbers in each line is constant, we may call this a magic pentagram. The magic pentagram is, however, not unique. Draw another magic pentagram

46. CROSS AND SQUARE



It is also possible to cut the cross into four pieces only which can be reassembled to form a square, as shown in the figure.

48. A BRIDGE PROBLEM



Here, we give a geometric proof that AP + PQ + QB is the shortest path. Take RS perpendicular to the banks at any other place for the bridge. Then it is easy to see that both PQBB' and RSBB' are parallelograms. Hence, BS = B'R and BQ = B'P. Now, AR + SB = AR + RB' > AB' = AP + PB' = AP + QB. Hence, AR + RS + SB > AP + PQ + QB, for RS = PQ. In case, the banks are not strictly parallel, the solution is more difficult and worth investigation by a mathematician. Here, some kind of restriction on the bridge will be necessary, say, that the bridge should be equally inclined to the two banks.

49. THE MISSING DIGIT

We said that the number your friend started with may have four or five digits. Actually, however, the number may have any number of digits except the trivial case of only one digit. If you want to know why the trick always works, you will first need a proof of what was stated in the Hints. Suppose we take a number of four digits, a, *b*, *c*, *d*. The number is actually $1000a + 100b + 10c + d$. If you subtract from this the sum of the digits $a + b + c + d$, you will get $999a + 99b + 9c$ which is obviously divisible by 9 and this was to be proved. Now, we know the rule of casting out nines. In effect, this implies that if a number is divisible by 9, the sum of its digits is also divisible by 9. That is why the trick will always work. There is only one

snag. If the digits given to you ultimately add upto 9, then you will announce that the digit struck out is 9 minus 9 which is 0. But that may be sometimes 9. So, in this case you must announce that the digit struck out is 0 or 9.

50. WATER AND WINE

It is immaterial how big or small the spoon is. After the two transfers, you may do two more transfers as before or indeed any number of pairs of transfers. In fact, you need not use a spoon at all. You may hold and tilt one of the glasses over the other and pour any amount of liquid in the other and then pour any amount from the second into the first and continue this mixing process as long as you like. The only important condition is that ultimately you must leave each glass exactly half-full. Then, as explained in the solution, the quantity of wine lost in the wine glass is to be found in the water glass and vice-versa.

51. MAGIC HEXAGON

The solution is unique. It is unique even if the six numbers as given are not given at all. But, to find this unique solution starting with all the cells empty appears to be difficult. That is why we gave those six numbers as help.

If you delete the 12 border cells, one can easily see that we cannot fill in the 7 cells with numbers 1 to 7 so as to have the magic property of a constant sum. On the other hand, we may add another round of 18 more border cells making a bigger hexagon but then it is not possible to fill them with numbers from 1 to 37 which will have the constant sum property. In fact, we can have bigger hexagonal pattern of cells by increasing the number of border cells (going outwards) and yet we will no more have the constant sum property. Indeed, it is difficult to prove that the only magic hexagon is the one given in our problem.

52. REMAINING REMAINDER

Here is the proof why the trick in our problem will always work. Let a,b,c be the digits your friend started with. Then, the three two-digit numbers are $10a + b$, $10b + c$ and $10c + a$ in this order. Let x, y, z, respectively be the remainders when these are divided by 13, so that we may write $10a + b = 13p + x$, $10b + c = 13q + y$ and $10c + a = 13r + z$. Now, $4x - 3y = 4(10a + b - 13p) - 3(10b + c - 13q) = (10c + a - 13r) + 13(3a - 2b - c - 4p + 3q + r) = z + 13m$, for some m which shows that if $4x - 3y$ is divided by 13, the remainder will be z.

An exactly similar problem can be formulated in which the divisor is not 13 but 7. In this case, if the remainders are, as

before, x, y, z in this order, we can find from two of them the third by the formula $3y - 2x = z$. For example, let us start with the three digits 2,5,4. Then, divide by 7, the numbers 25, 54, 42; the remainders are 4, 5, 0 respectively. Now, 0 is also the remainder when you divide $3 \times 5 - 2 \times 4$ by 7. The proof is on the same lines as above and is left to the reader.

We do not propose the question: How does one find such formula ? If you are innovative enough, you will find it interesting to find the formula when the divisor is 11.

53. A MULTIPLE OF 11

Can you find in a similar way the smallest multiple of 11, using nine different digits ? Note, here, that zero can be one of the digits, which means that you should drop an appropriate non-zero digit, unless of course, you change the problem itself saying that all the ten digits should be used.

56. HOW MANY SQUARES

We may generalise the question. If a rectangle of length m units and width n units is divided up into mn small unit squares, the total number of squares of all sizes will be, by the same kind of argument, as given in the solution,

$$mn + (m-1)(n-1) + (m-2)(n-2) + \dots \quad + (m-n+1)(1).$$

In particular, if the given figure is itself a $n \times n$ square, the total number of squares in it will be $n^2 + (n-1)^2 + (n-2)^2 + \dots + 1^2$ $= \frac{1}{6} n(n+1)(2n+1)$ by a known formula.

Ask a friend how many squares big and small are there on an ordinary chess board. Long before he comes out with his answer, you will be ready with the correct figure, namely $8^2 + 7^2 + \dots + 2^2 + 1^2 = \frac{1}{6} \cdot 8 \cdot 9 \cdot 17 = 204$. Read this question : A rectangle is intersected by any m lines parallel to the length and any n lines parallel to the width. How many rectangles of all sizes are there in the figure ? I have found many students perplexed by this question. Actually, however, it is easy. A rectangle is formed by some two lines parallel to the length and some two lines parallel to the width. In the direction of the length, there are in all $m + 2$ parallel lines and you can choose two from them in $\frac{1}{2}(m+2)(m+1)$ ways. And two lines from $n+2$ parallel lines in the other set can be chosen in $\frac{1}{2}(n+2)(n+1)$ ways. So, the required total number of rectangles is simply $\frac{1}{4}(m+1)(n+1)(m+2)(n+2)$.

59. PROFIT AND LOSS

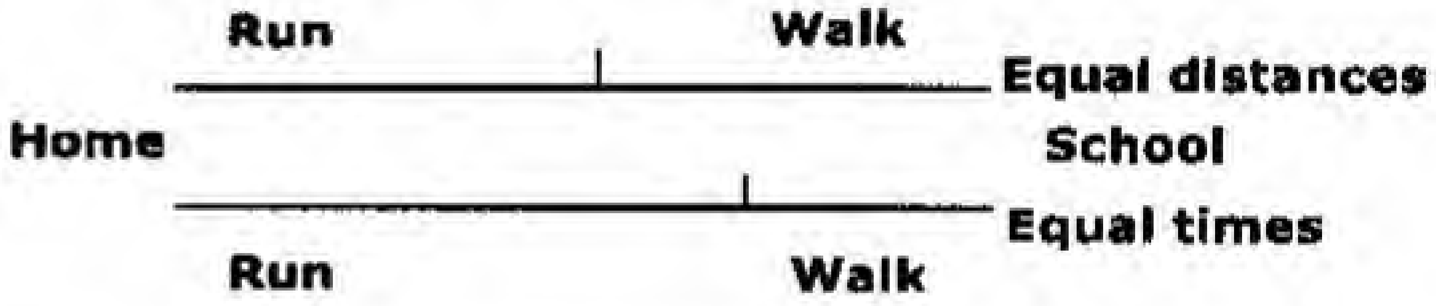
It should not be surprising that though the selling prices are the same and the profit and loss are both 10%, they do not cancel each other. Because, the percentage is calculated on the cost prices, not the selling prices. Had the cost prices been the same, equal profit and loss will cancel each other.

It may be noted that the overall loss will be 1% irrespective of what the equal selling prices are, not necessarily Rs. 1584. It is only dependant on the equal profit and loss percentages, which is 10% here. You may, pehaps, be intersted in proving the following :

I bought two articles at different prices and after some time sold them for some equal prices, gaining x% in one case and losing x% in the other case. Show that, on the whole, I suffered a loss of $\left(\frac{x^2}{100}\right)$%. In our particular problem, the loss is $\left(\frac{10^2}{100}\right)$% or 1%.

61. HALF DISTANCE-HALF TIME

This should be obvious without algebra. It is just common sense : if times taken for running and walking are same, then, obviously, distance covered in running is more than in walking. But the total distance covered is the same in both cases. So, a certain common distance is walked in the first case and run in the second case. So, the second case is to be preferred.



62. PERFECT SQUARE

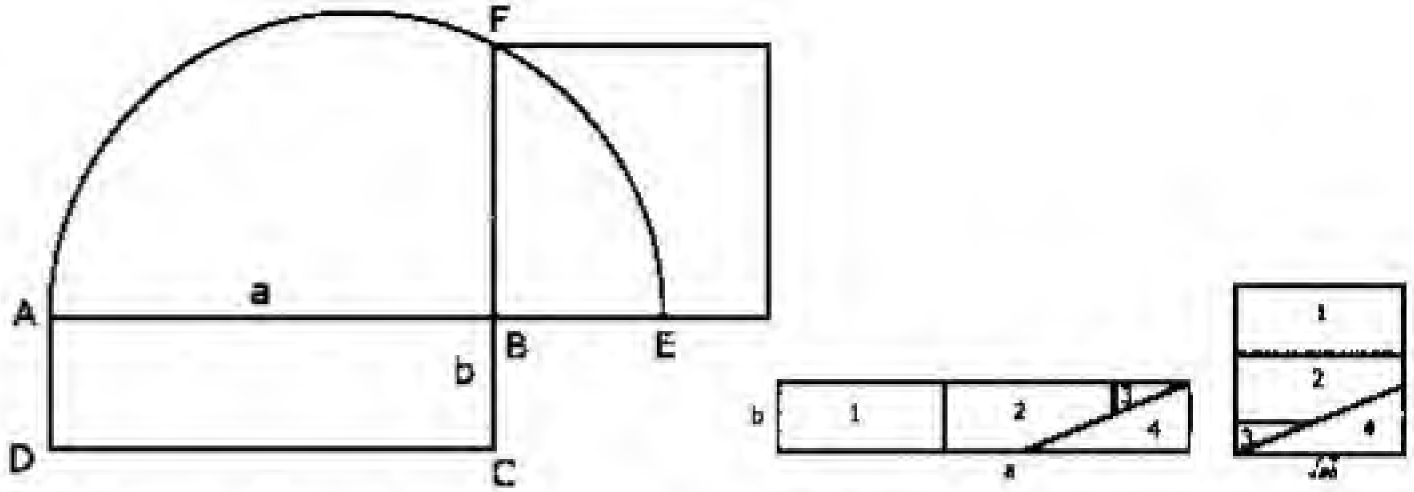
The work will be simpler, if you know the process of finding square roots directly in base seven, as shown below, without conversion into decimals :

2)	$\dot{6}6\dot{6}6\dot{6}$ (243		243
	4		× 243
44	266 ; 4		1062
	242	—	1335
513	2466 3		516
	2142		66342

63. RECTANGLE - SQUARE

To get a side of the square equal in area to a rectangle, by a geometrical construction is easy as shown here. Draw a segment AE of length $(a + b)$ and take B in it so that AB = a. On AE as diameter, describe a semicircle. Draw a perpendicular at B to meet the semicircle in F. Then, BF is a side of the required square and is of length $\sqrt{ab}$. Proof of this is well-known.

It is necessary to note that the dissection given in the solution is possible only if $a < 4b$. ($a = 4b$ is a trivial case, as a single middle cut will give two pieces, one of which can be placed over the other.) If $a > 4b$, but $< 9b$ then the rectangle will have to be cut into four pieces, one of which is a rectangle of length $\sqrt{ab}$ and width b and the other three can be obtained as before.



Generally, if $a > n^2b$ but $< (n + 1)^2 b$, where n is any natural number, the minimum number of pieces which will convert the rectangle into a square is $(n + 1)$, of which $(n - 2)$ will be rectangles of length $\sqrt{ab}$ and width b.

64. RULER CONSTRUCTION

Students of mathematics may ask for a proof. Here, we give. Refer to the figure given in the solution. Obviously, to prove that $YB = \frac{1}{3} AB$, it is enough to show that $MY = \frac{1}{2} YB$.

By Ceva's Theorem, $\frac{MY}{YB} \cdot \frac{BL}{LP} \cdot \frac{PK}{KM} = 1$

By Menelaus' Theorem, $\frac{AM}{AB} \cdot \frac{BL}{LP} \cdot \frac{PK}{KM} = 1$

Comparing these two results, $\frac{MY}{YB} = \frac{AM}{AB} = \frac{1}{2}$

65. COLOUR CUBES

Can the following two problems be solved on the same lines?

(a) In how many different ways can a cubical frame be made using for the edges, 12 differently coloured rods of equal length ?

(b) In how many different ways can 8 differently coloured tiny balls be placed at the vertices of a cube ?

You will be surprised to know that there are only two different ways of painting the four faces of a regular tetrahedron with four given colours.

You may innovate many other problems of colouring the faces of some polyhedra. They may be difficult to solve, unless the solver uses his ingenuity.

66. MAGIC OCTAHEDRON

A variation of the same problem is as follows : Write numbers 1 to 8, one at each of the eight corners of a cube so that the sum of the numbers at the four corners of each face is the same.

67. RECTANGULAR MESH

A number of points, some pairs of which are joined by lines is a network or a graph. The foundation for the study of graphs was laid by L. Euler. The point from where an odd number of lines emanate is called an odd node and even number of lines, even node. Euler showed that in a graph or a design, if the points are all even nodes or there are at most two odd nodes, it can be drawn unicursally, i.e. without lifting the pencil off the paper or tracing a line already drawn. He also showed that the number of odd nodes in a design is always even. If this number is $2k$, the design will consist of k circuits, i.e., the pencil will have to be lifted off the paper $(k-1)$ times.

Our design has ten odd nodes. So, it can be drawn in five circuits, i.e., the pencil may be lifted four times only.

68. KÖNIGSBERG BRIDGES

It was again L. Euler who first showed that it is impossible to walk over each bridge once only. It goes to Euler's credit that he was the founder of the Network Theory or the modern Graph Theory. For an interesting history of the problem, the reader is referred to any book on Mathematical Recreations.

70. DIGIT PAIRS

In the solution, we have proved that it is impossible to find a sequence of 10 digits consisting of pairs of 1's, 2's, 3's, 4's, 5's possessing the property in question. The general question would be:

For what values of n, there exists a sequence of $2n$ digits consisting of pairs of 1's, 2's, ..., n's, such that for every p ($\leq n$), there are p digits between the two p's ?

We have seen that the answer is 'yes' for the values 3 and 4 of n, and 'no' for the value 5. The answer is 'no' for $n = 6$, for which the proof is not difficult. Surprisingly, the answer is again 'yes' for

$n = 7$ and 8. The sequences are

57236253471614

7385236275481614

It can be shown that the sequences exist if n is of the form $4r$ or $4r + 3$ and do not exist if n is of the form $4r + 1$ or $4r + 2$. We have with us sequences for $n = 11, 12, 15, 16$. We do not give them here, but urge the reader to find them.

71. 63 = 64 = 65

This problem has something to do with the well-known Fibonacci Sequence :

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...

in which each term (after the second) is the sum of the two previous terms. This sequence has innumerable interesting properties. One of the properties is : The square of any term differs from the product of its two adjacent terms by unity. If we take the sixth term, we get 8^2 differs from 5 x 13 by 1. This 1 is small compared to 8^2 or 64. It is this which caused the illusion. You may, now, do a similar experiment, using the fact that 13^2 differs from 8 x 21 by 1. In fact, in this case, the illusion will be more mystifying.

72. CONSTANT SUM

How were those ten numbers, five in the top row and five in the extreme left column found ? They were not found by the author, but the solver has to find them. If you desire to have a problem like this of your own, you should proceed as follows : First, draw up a 25 squares mesh with all the cells empty. Choose any ten numbers arbitrarily and write five of them in top row above the mesh and five in the extreme left column

outside the mesh. Then, fill up the 25 cells, the number in each cell being the sum of two of the chosen numbers, as explained in the solution.

After this, wipe out the ten chosen numbers and you will have your puzzle ready for presentation to a friend.

It is not necessary, but very desireable to make the 25 numbers all different. During construction, some two or more numbers may become equal. To avoid this, alter suitably or adjust the numbers chosen, though still they are arbitrary.

There is nothing special about 25. You can have a 16-square or 36-square mesh or a larger mesh.

73. SQUARE HOLE IN A SQUARE

The genesis of this puzzle is Pythagoras' Theorem. For, obviously, if you construct a right-angled triangle whose sides are the side of the square you started with, and the side of the square hole, then the hypotenuse will be the side of the bigger square. Indeed, this is an excellent generalisation of the famous 'dissection proof' of Pythagoras' Theorem, by Perigal.

74. FOUR 4'S

This problem is a favourite of many solvers who spend a lot of time on this for the sheer fun of it. This is mentioned in many books on Mathematical Recreations and magazines. There are people who are crazy enough to have found expressions for all numbers from 1 to 1000 using only four 4's and arithmetical signs. There are others who find several expressions for the same number. For instance,

$$15 = \frac{44}{4} + 4 \qquad = 4 \times 4 - \frac{4}{4} \qquad = \frac{4 + 4 - \sqrt{4}}{.4}$$

$$= \frac{4}{.4} + \frac{\sqrt{4}}{.4} \qquad = \frac{\sqrt{4} + .4}{.4 \times .4}$$

Among the arithmetical signs used, you may include the factorial sign (!). Note that 4! = 24 and the recurring decimal

$.\dot{4} = \frac{4}{9}$. So, 15 can also be written as

$$= \frac{\sqrt{.\dot{4}} + 4}{.4 \times .\dot{4}} \qquad = 4! - \frac{4 - .4}{.4}, \text{etc.}$$

75. SEVENTEEN DOMINOES

Instead of filling the cells with numbers, you may colour them as in a chessboard alternately black and white (as is done in some books). A domino, placed anywhere (vertically or horizontally) on the board, will always cover one black and one white cell.

So, if the two blocked cells correspond to different colours, then the problem is solvable, while if they correspond to the same colour, the problem is unsolvable.

Instead of using 36 square mesh, you may use a 64 square mesh. The number of squares on a side must be even. Why ?

76. EIGHTEEN DOMINOES

If you use a square mesh of 8 x 8 = 64 cells, it is possible to cover it up with 32 dominoes so that no straight line can be drawn from one side to the opposite side without cutting a domino. Try to do this. Where does the argument given in the solution fail here ?

77. FIVE BY FIVE COLOUR SQUARE

The five colours are designated by the five letters A, B, C, D, E. You cannot expect these letters to be properly oriented in

the solution. This difficulty will not arise, if you actually make or imagine a thick paper or card coloured model.

We also have an easier puzzle : Four by Four colour square in which two trominoes and five dominoes are used.

79. TWO FRIENDS

After disappointment at not meeting their friends at their houses, suppose each waits for 5 minutes and starts returning home, at what time will they meet again on the road ?

80. PAPER FOLDING

In the problem, the protruding parts of the strip are of lengths *a* and *b* at the two ends. If they are *a* and *b* at the same end, what will be the distance between the two creases ?

81. TRIANGULAR TIER

In the problem, each side of the triangle is divided into ten equal parts. The result can be generalised. If each side is divided into *n* equal parts, the total number of triangles of all sizes that are formed is given by

$\frac{1}{8}(2n^3+5n^2+2n)$ if n is even.

and $\frac{1}{8}(2n^3+5n^2+2n-1)$ if n is odd.

Note : whether n is even or odd, both the formula can be combined into a single formula

$$\frac{1}{8}\left[2n^3+5n^2+2n-\frac{1}{2}+\frac{1}{2}(-1)^n\right]$$

Students of mathematics are urged to obtain this result.

More enterprising students may obtain further the following result.

The formula for the number of parallelograms of various sizes is $\frac{1}{8}\ n(n+2)(n^2-1)$. There is only one answer, whether n is odd or even.

Note that the triangle need not be equilateral for obtaining these formulas.

If, however, the triangle is equilateral, then obtain the following results:

The number of rhombi of various sizes is

$\frac{1}{8}n(n+2)(2n-1)$ if n is even

and $\frac{1}{8}(n^2-1)(2n+3)$ if n is odd.

We can give a single answer whether n is odd or even, namely

$$\frac{1}{8}\left[2n^3+3n^2-2n-\frac{3}{2}+\frac{3}{2}(-1)^n\right]$$

The number of regular hexagons of various sizes is given by

$\frac{1}{18}(n^3-3n)$ if n is of the form $3m$

$\frac{1}{18}(n^3-3n+2)$ if n is of the form $3m+1$

$\frac{1}{18}(n^3-3n-2)$ if n is of the form $3m+2$

It is interesting to note that all the three formula can be combined into one. The answer is

$$\frac{1}{2}\ m(n^2-3mn+3m^2-1)$$

where $3m$ is the largest multiple of 3 not exceeding n.

What we have not given, leaving it as an exercise, is the number of trapezia of various sizes.

83. 1 TO 80

Note that there is no number common to the two sides of each card. If you can answer the following questions, you will be able to probe the thoughts of the author who innovated the puzzle.

(1) Why are there 27 numbers on each side of the card ?

(2) What is special about 80?

(3) What is the property of the number chosen by the subject, so that he presents one, two, three or four cards?

(4) Is the distribution of numbers in each face of each card unique ?

85. LAST THREE DIGITS

If four, five or more last digits in the product of two numbers and one factor are given, can you not, now, find the same number of last digits in the other factor ?

86. BISECTION TIME

Can you find out how many times does the minute hand stand equally inclined to the vertical and the hour hand, in a twelve hour period ?

87. CALENDAR PUZZLE

Instead of the calendar, consider a rectangular mesh (or matrix) of numbers of any arithmetical progression in l rows and m columns. Prove that the value of a determinant of any minor of the mesh is divisible by m. This is a generalization of the problem in which the AP is of consecutive natural numbers and $m = 7$.

88. SQUARE YEAR

We saw in the solution that the first century which will be square free is the 26th. After this, some subsequent centuries will have square years in them. Can you find out when the next square free century will occur ? In fact, after a very long time, any given number of consecutive square free centuries will occur. Can you work out the pattern of occurrence of square free centuries? But, centuries with square years will not cease to appear. They will occur eternally, even after billions of years. However, they will be scarcer and scarcer, separated by enormous gaps of square free centuries ! Do you see why?

89. HOW MUCH AREA ?

The area of the annular region is always the same as long as the length of a side of the regular polygon remains the same, and is independent of the number of sides of the polygon.

91. IS IT A SQUARE ?

In obtaining the digital root of a number, when you add the digits, you may always ignore, i.e. drop the digit 9 or digits which total 9, like 4, 5; 6, 3 etc. This is known as "the Rule of Casting out nines". Do you see the reason for this ? Do you also see the reason why it is true that the digital root of a product is the same as the digital root of the product of the digital roots of the factors ?

92. HANGING ROD

What we have actually done here is equating the moments of the two weights about B.

93. (10 – 3) CONFIGURATION

The configuration shows symmetrical properties of incidence of points and lines. But, the figure arose from a pair of triangles ABC, PQR where the lines joining the corresponding vertices, namely, AP, BQ, CR are concurrent at O and the theorem says that the intersections of the corresponding sides (BC, QR), (CA, RP), (AB, PQ), namely X, Y, Z are collinear. However, you can see that the same figure can arise from the pair of triangles RYX, OAB with RO, YA, XB concurrent at C and the intersections of (YX, AB),

(XR, BO), (RY, OA), namely Z, Q, P collinear. That is, you realise the same property in the figure in two different ways. Actually, it is amazing that you can realise the property in ten different ways !

Geometricians have found many more interesting properties of this configuration. Did you notice that we have not said anywhere that the whole figure should be in one plane? Indeed, the figure need not be in one plane. Can you visualise such a configuration in space?

94. DIVISIBILITY TEST

The divisibility tests for other divisors are as follows :

Divisor	Test
2	Last digit must be 0, 2, 4, 6 or 8
3	Sum of the digits must be divisible by 3
4	The number formed by the last two digits must be divisible by 4
5	Last digit must be 5 or 0
6	Both the tests for 2 and 3 must hold

8 The number formed by the last three digits must be divisible by 8

9 The sum of the digits must be divisible by 9

10 Last digit must be 0

11 The sum of the digits in the odd places must differ from that in the even places by zero or a multiple of 11.

School children know all these tests. But, they miss the test for the divisor 7. That is why we have given it here. Some may remark that actual division by 7 is as simple as using the test. True. But, one should look at the test not from a practical point of view, but as an elegent and interesting property of numbers.

For the divisor 12, combine the tests for 3 and 4.

We give here the test for the divisor 13. Remove the last digit from the given number N and to what remains, add four times the digit removed, and get N_1. Then from N_1 get N_2 in the same way. Continue this process until you get a small number which can be easily tested for divisibility by 13. Prove that this will always work.

A general test for any prime number as divisor is known. We would urge any intelligent student of mathematics to discover it.

96. PERPETUAL CALENDAR

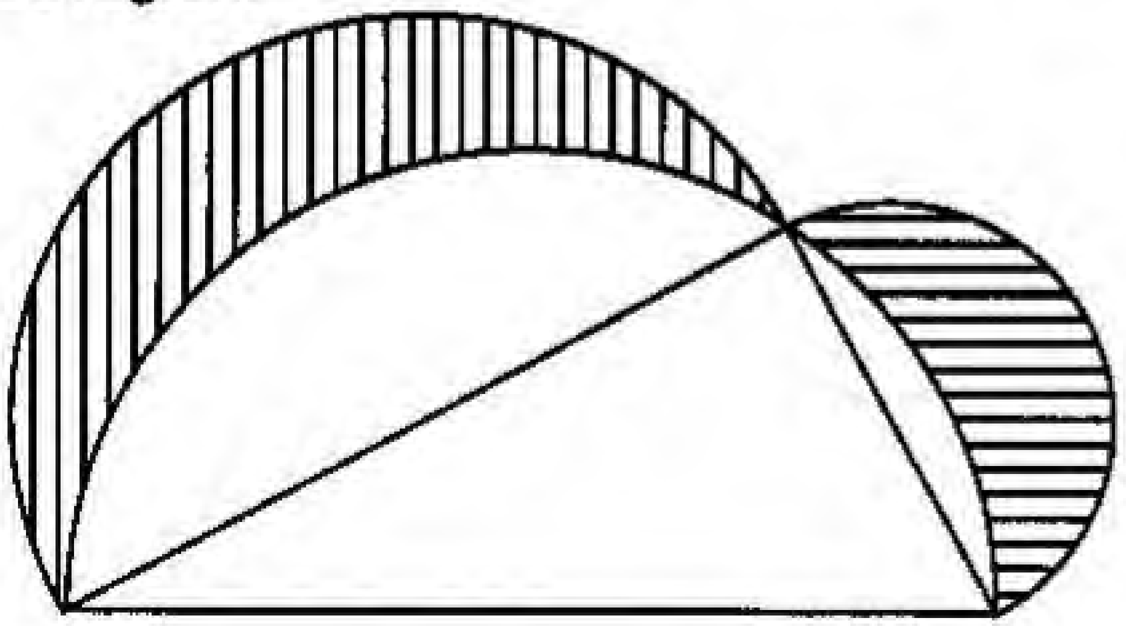
The calendar we have given is perpetual in the sense that it will hold good for any date in the past or future. We had assumed that the year number contains four digits. However, if a future year number contains more than four digits, retain only the last four digits, ignoring the other digits. This will work. If you are using a year in the past, you should know a historical fact. The calendar we are using is known as Julian Calendar modified as Gregorian Calendar. Owing to certain astronomical reasons, errors were detected in the reckoning of the calendar days and in 1752, by a parliamentary act in England, ten days (Sep. 3 to Sep. 13) were dropped. There was a great uproar and confusion on this account everywhere. See Encyclopedae Brittanica under Gregorean Calendar and an interesting article on "Change of style" in Lobbon's English Essays. So, in using our formula for a date before Sep. 13, 1752, this fact must be taken into account.

97. CUT NUMBER

Any integer p, which will make $8p^2 + 8p + 1$ a perfect square q^2, will give a cut number, $q + 2p + 1$. But, this is not of much help, because, we do not know how to find such a p. Of course, you may verify that $p = 2$ and $p = 14$, will make $8p^2 + 8p + 1$ a perfect square. If you are inquisitive to know the third cut number after 6 and 35, it is 204. If you are an adept in number theory, you can certainly work out a theory of cut numbers.

98. A CURIOUS RESULT IN AREAS

If you take the other semicircles on the sides of a right angled triangle, you will get the figure shown here. This is a very familiar figure. The area of the triangle is equal to the area of the shaded region.



99. A HOLE THROUGH SPHERE

This is unbelievably true ! Even if you take a very huge sphere, say, the Earth (6400 kms radius) and scoop out a cylinder of height 12 cm only, what will remain will be 288π (a little more than 800) cubic centimeters of earth only !

100. AVERAGES

Instead of taking all pairs from any set of n numbers, if we take all subsets of r numbers ($r < n$), take the average of each subset and then the average of these averages, you will get the original average of the n numbers. Is it difficult to prove this ?

101. MEANS AND MISSING NUMBERS.

You could have taken x in any other empty cell, and you will get the same answer. The above solution is unique. Prove that the sum of the three numbers initially given in any puzzle of this kind, is thrice the central number. Also prove that in any puzzle of this kind, the sum of the numbers in the two pairs of opposite corners of any square are equal.

102. AREA OF THE FIELD

In any such problem, can you see that the products of diagonally opposite figures must be equal? Use this to get

the solution even quicker. For instance, $24 \times 60 = 32 \times A$ gives $A = 45$.

103. WEEK DAY REPEAT

In the comments on problem 96, we had said that if the year number contains more than 4 digits, retain only the last 4 and ignore the other digits, without explaining how this will always work. Now it is clear. Because ignoring those digits amounts to ignoring multiples of 10,000 which are multiples of 400 also.

105. SQUARE WITHIN A SQUARE

Create a similar problem for equilateral triangle inside an equilateral triangle (not 1.6 times).

106. COMPARISION OF AREAS

Very interestingly, even if we take any number of circles in the two squares, say m and n, the two areas will be "equal".

107. A CHARACTERISTIC OF 4 x 4 MAGIC SQUARES

It will not be difficult to show that the sum of the four entries described below is S in every case.

(i) The four corner squares.

(ii) The two middle squares in the top row and bottom rows.

(iii) The two middle squares in the left and right columns.

In a special 4 x 4 magic square, there are many other patterns of four entries whose sums are always S. To show this, we have a gadget, in our laboratory, known as 'versatile magic square'.

108. SERIES SUMMATION

Find the number of terms to be taken of the series $1 + 2 - 3 - 4 + 5 + 6 - 7 - 8 + 9 + 10 \ldots$ to get the sum equal to 2007.

111. ROD ON A SEGMENT

If you have solved the problem without using algebra, hearty congratulations!

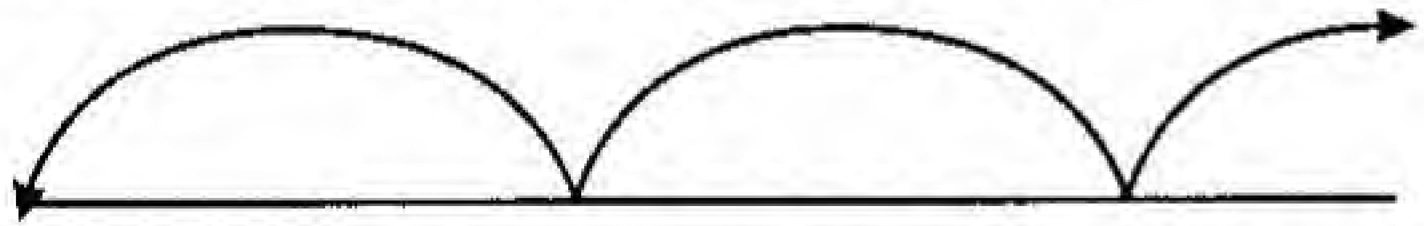
112. SUMS OF SQUARES

You have been asked to supply two other examples, or at least one more. Actually there are an infinity of examples each starting with a composite odd number. So this must be easy. Now we asked you to give an example of a number which can be expressed as a sum of three squares in two different ways. Here is one example:

$381 = 10^2 + 5^2 + 16^2 = 19^2 + 4^2 + 2^2$.

113. ROLLING DISC

If the disc, instead of rolling inside a circle, rolls, without sliding, on a straight line, a point on its edge will describe a beautiful curve called cycloid, consisting of a sequence of arches.



On the other hand, if the disc rolls inside or outside circles of various other discs, we get (a number of) closed curves called hypocycloids and epicycloids consisting of beautiful arches or loops. The study of these curves belongs to higher mathematics and is beyond the scope of this book.

114. MULTIPLES OF 11

If we add one more digit, that is, if we use digits 1, 2, 3, 4, 5, 6, 7 all, but each only once, you will be surprised to know that now the number of multiples of 11 is 576. Proof may not be difficult but a little tedious. Try.

115. PROBLEM OF FOLDING A RECTANGULAR PAPER

If the paper is folded about BD bringing the two triangles in one plane and C" is the new position of C, calculate the lengths of CC" and AC" .

Generalise the problems by replacing 20 units and 15 units by *a* units and *b* units respectively.

116. NUMBER OF PRIZES

Is it possible to partition a number, not necessarily 33, into five parts satisfying similar conditions ?

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- *Brain Sharpeners*
- *Our Solar System*
- *Seeing is not Believing*
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- *Environment and My Life*
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- *Shikhvano Anand Teachers Manual - 1*
- *Shikhvano Anand Teachers Manual - 3*
- *Vaignanik Dr. Vikram Sarabhai*

A. R. Rao

Professor A. R. Rao, born in 1908, has had a distinguished career spanning almost seven decades. He has been a professor of mathematics at many colleges in the state of Gujarat and was also the Principal of the Sir P P Institute of Science, Bhavnagar.

At VASCSC, Prof. Rao was the Principal Mathematician from 1974 till 1988. After 1988 till the present he is a Mathematics Consultant.

Prof. Rao has held many prestigious posts and has been awarded and felicitated by numerous academies, societies and at conferences for his creative mathematical abilities, lectures, development of innovative mathematical puzzles and teaching aids.

Two documentary films have been produced depicting Prof. Rao's life, work and achievements. He also has a number of publications to his credit in English and Gujarati.

In 1998 Prof. Rao was awarded a National Award for the best effort in the country for popularization in Science and Mathematics, by the National Council of Science & Technology Communication, (DST), Government of India. In 2008,he was honoured by the Government of Gujarat and recently, in 2010, he was awarded the "Harmony Silver Award" by the Harmony Foundation, Mumbai.

Prof. Rao, passed away on 11 April 2011 at the age of 102. Until then, he was active in the popularization of mathematics and in the development of new innovative educational aids and mathematical games.

You cannot perform this trick unless the dice are normal or you know that the dice are righthanded or left-handed. Also, instead of two dice, you may have any number of dice stacked one over the other with the top face of the top dice covered. Try to find the hidden faces shown in the illustration given here.

41. DIGITS IN A TRIANGLE

There is no mathematical method here. The problem is meant to test your keen sense of observation. Now that you know the patterns in the two given schemes, can you find the pattern in the third scheme given below?

2 1 6 3 5 4

5 5 1 4 3

6 1 6 6

2 5 3

3 0

3

42. UNICURSAL DESIGN

It is not necessary to remember the entire sequence of

letters and suffixes given in the solution. One may only remember the sequence ACEBD to draw this particular design. Observe that ACEBD is cyclically repeated nine times and the suffixes 1 to 9 also cyclically repeated five times. Note the order ACEBD is arbitrarily chosen. You can get another design by choosing some other order of the five letters, say ADEBC. Try. Thus there are a large number of designs like this. Which of these is most pleasing will depend on your personal aesthetic sense. Draw a few of them and pick up one for drawing with "rangoli powder" at your doorstep as a part of festival decorations.

There is scope for further variations. Here, we have taken sets of five points in each of nine radial lines, to begin with. Instead, to get a small design you may take sets of four points in each of seven radial lines. You will get a bigger design if you take sets of seven points in each of eight or ten radial lines. Try some of these variations. The effort may be rewarding.

Sometimes, the design may not come out in a single circuit (i.e. not unicursal) if all the points are to be utilised. The complete design will consist of two or more circuits and all the circuits in the same design will be congruent. Apart from artistry, a mathematician might like to do a small piece of